

## BOUNDS ON NEUTRINO MASSES FROM NEUTRINO DECAY RATES, COSMOLOGY AND THE SEE-SAW MECHANISM\*

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We study the constraints imposed on the masses of  $\nu_e$ ,  $\nu_\mu$  and  $\nu_\tau$  on the basis of direct experimental bounds, cosmological bounds, theoretical calculations of neutrino decay rates, experimental bounds on related decays of charged leptons and the structure of neutrino mass matrices in the “see-saw” mechanism. We consider standard model amplitudes as well as contributions from all “beyond standard” models. Assuming a simple “reasonable” form of the see-saw mechanism, we derive the bounds:  $m(\nu_\tau) \leq 65$  eV,  $m(\nu_\mu) \leq 4$  eV,  $m(\nu_e) \leq 0.02$  eV,  $M(W_R) \geq 50$  PeV. Possible ways of evading these bounds are discussed.

### 1. Introduction

If the known left-handed neutrinos are *exactly* massless, neutrino physics becomes relatively simple. There are no neutrino oscillations, no mixing among different generations, no neutrino decays, no mass pattern to explain, no interplay between Dirac and Majorana masses, etc. There is “only” one problem: We must find some *exact* symmetry which prevents the neutrinos from acquiring masses to all orders in the standard model as well as in the presence of all possible new physics effects which go beyond the standard model. No such principle is known and most theories beyond the standard model actually allow a variety of contributions to the neutrino mass. Consequently, it is widely believed that neutrinos *do* have masses. In this paper we assume that neutrinos are light, *but not massless*.

If the neutrinos *do* have masses, we immediately face a long list of new questions. We may have right-handed neutrinos with new interactions. The number of fundamental parameters of the standard model increases. We have not only unknown masses but also Cabibbo-like generation mixing among leptons and at least one KM phase leading to *CP* violation. All the unsolved problems concerning the masses and mixing angles of quarks and charged leptons, suddenly arise in the neutrino

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sector. We are likely to have neutrino oscillations in vacuum, possibly enhanced in matter. Neutrinos are likely to decay in various ways, raising the question of their lifetimes and decay branching ratios. We may have both Dirac and Majorana mass terms, leading to neutrino mass matrices even in the case of one generation and to interesting mass patterns in the realistic case of several generations.

The existing experimental information on all of these issues consists only of upper limits. There is no conclusive evidence for neutrino masses, mixing, oscillations or decays. No one has seen evidence for right-handed neutrinos, Majorana masses or  $CP$  violation in the leptonic sector.

The present direct limits on the masses of the three known left-handed neutrinos are [1–3]:

$$\begin{aligned} m(\nu_e) &\leq 18 \text{ eV}, \\ m(\nu_\mu) &\leq 250 \text{ keV}, \\ m(\nu_\tau) &\leq 70 \text{ MeV}. \end{aligned} \tag{1.1}$$

It is very clear that these neutrinos are much lighter than the corresponding quarks and charged leptons in the same generations. How can we explain this fact? In principle, the Dirac masses of the neutrinos are free arbitrary parameters of the standard model. In order to “account for” the tiny  $\nu_e$  mass, all we have to do is to declare that the single Higgs doublet of the minimal standard model couples to  $\nu_e$  with a Yukawa coupling which is smaller than  $10^{-10}$ . This would be unsatisfactory for *two* independent reasons. We do not understand the reason for such a small Yukawa coupling and, even if we did, we do not know why it applies only to neutrinos and not to any other fermions in the standard model. Such a situation cannot be rigorously ruled out but it would be extremely unnatural and *we assume that it does not occur*.

An alternative possibility is to assume that left-handed neutrinos have only Majorana masses which are due to their *direct* coupling to a Higgs *triplet* carrying two units of lepton number. However, in order for these masses to be extremely small we must have either a tiny vacuum expectation value for the Higgs triplet or, again, extremely small Yukawa couplings. Such a possibility is as unlikely and unnatural as the previous one and *we assume that it does not occur*.

There must be a good explanation for the fact that left-handed neutrinos are much lighter than all other fermions. Fortunately, we have a general mechanism which can lead to such an explanation. Assuming that we have some physics beyond the standard model and that it corresponds to a new energy scale  $\Lambda \gg M_W$ , we may describe its low energy effects in terms of an effective lagrangian. Such a lagrangian may include a dimension-five term of the form

$$\frac{h}{\Lambda} \phi \phi \nu_L \nu_L, \tag{1.2}$$

where  $\phi$  is the usual Higgs doublet of the standard model,  $\nu_L$  is any left-handed neutrino and  $h$  is an unknown dimensionless effective coupling constant. With the usual symmetry breaking of the standard model, such a term would yield a neutrino Majorana mass of the general order of magnitude of:

$$M(\nu) = \frac{h \langle \phi \rangle^2}{\Lambda}. \quad (1.3)$$

Ordinary fermion masses are of order  $h' \langle \phi \rangle$ , where  $h'$  is the usual Yukawa coupling of  $\phi$  to fermions. It is therefore clear that the resulting neutrino (Majorana) mass is substantially smaller than the normal (Dirac) mass of an ordinary fermion (quark or charged lepton) by a ratio:

$$\frac{M(\nu)}{M(f)} = \frac{h}{h'} \frac{\langle \phi \rangle}{\Lambda} \ll 1. \quad (1.4)$$

For sufficiently large  $\Lambda$  we can easily get extremely small neutrino masses.

The above scenario is sufficiently general to accommodate a wide variety of theoretical ideas which go beyond the standard model. The only crucial ingredient is the existence of a new energy scale  $\Lambda$  and the ability of the new physics to induce an effective term of the necessary form (or, possibly, a higher dimension term yielding a neutrino mass which is inversely proportional to a higher power of  $\Lambda$ ). The best known realization of the above mechanism is the “see-saw” matrix [4] for neutrino masses.

In this paper we assume that the general mechanism responsible for the small neutrino masses is of the type described above. We combine information coming from cosmological considerations, theoretical particle physics arguments and indirect relations to experimental measurements of other processes in order to set severe upper bounds on the masses of the three known neutrinos.

Our analysis runs along the following lines [5]: Cosmology leads to a well known bound on the masses of *stable* neutrinos [6,7]. However, neutrinos with masses (except the lightest one) are practically certain to be *unstable*. In that case, cosmology provides us only with a relation between the mass and the lifetime of the unstable neutrino [8]. Such a relation, by itself, cannot exclude any neutrino mass-value. However, for any given neutrino decay-mode in any given model (“standard” or “beyond standard”) we may derive additional relations between the mass and the lifetime of the decaying neutrino. By combining the cosmological and the particle-physics constraints for the decay modes of the same neutrino we may then be able to exclude certain mass ranges and to derive strong bounds on the neutrino mass.

Neutrino decays *within* the standard model as well as *some* neutrino decay modes in *some* “beyond standard” models were discussed by earlier authors. In this paper

we attempt to complete the discussion for all decay modes in all “popular” classes of theories which go beyond the standard model. For completeness, we briefly review previous results and combine them with our own results in order to draw our final conclusions. We find that, under very reasonable assumptions which we clearly state, *the masses of  $\nu_\mu$  and  $\nu_\tau$  must be smaller than 65 eV*. Under slightly stronger assumptions we further conclude that  $\nu_\mu$  is probably lighter than 4 eV and  $\nu_e$  is lighter than 0.02 eV. We also conclude that the scale  $\Lambda$  responsible for the neutrino masses is probably above 50 PeV (1 PeV =  $10^3$  TeV). These results are significant as they improve upon the *direct* experimental limits by three to six orders of magnitude.

The structure of this paper is as follows: In sect. 2 we rederive the cosmological bounds on neutrino masses and lifetimes. In sect. 3 we introduce the see-saw mechanism and set our notations for the neutrino mass matrices and mixing angles. Sects. 4–10 are devoted to the study of neutrino decays within the framework of several classes of models. The case of the standard model is described in sect. 4. In sect. 5 we set the stage for discussing neutrino decays “beyond the standard model” and briefly mention some models which introduce extremely high energy scales. We continue with left-right symmetric (LRS) models (sect. 6), horizontal symmetries (sect. 7), substructure (sect. 8), supersymmetry (sect. 9) and models with spontaneously broken global symmetries (sect. 10). All models of sects. 4–10, together with the cosmological constraints of sect. 2, lead to strong bounds on the neutrino masses. In sect. 11 we study the see-saw mechanism in some detail, introduce a so-called “reasonable see-saw” assumption and show that it implies even more stringent bounds on neutrino masses. The combined information obtained in sects. 2–11 gives:

$$m(\nu_\tau) \leq 65 \text{ eV}, \quad m(\nu_\mu) \leq 4 \text{ eV}, \quad m(\nu_e) \leq 0.02 \text{ eV}. \quad (1.5)$$

In sect. 12 we study the implications of the cosmological bounds on the decays of hypothetical fourth-generation leptons. Finally, in sect. 13 we discuss our conclusions on neutrino masses, and their implications on the scale of physics beyond the standard model.

## 2. Cosmological bounds

### 2.1. COSMOLOGICAL PARAMETERS

Massive neutrinos contribute to the energy density of the universe. The requirement that this contribution should not exceed the present energy density of the universe, excludes a certain range of masses for stable neutrinos [6, 9, 7], and defines an allowed range for the mass and the lifetime of unstable neutrinos [8, 10]. There are two cosmological parameters that determine these limits: the Hubble parameter  $H_0$ , and the present energy density of the universe  $\rho_0$ . Equivalently, we can use the

two parameters  $h$  and  $\Omega$  defined by:

$$\begin{aligned} H_0 &\equiv h \cdot 100 \text{ [(km/sec)/Mpc]}, \\ \rho_0 &\equiv \Omega \rho_c. \end{aligned} \quad (2.1)$$

$\rho_c$  is the critical density, corresponding to a flat universe:

$$\rho_c = \frac{3H_0^2}{8\pi G} = 1.1 \times 10^4 h^2 \text{ [eV/cm}^3\text{]}. \quad (2.2)$$

Current estimates of  $h$  and  $\Omega$  are [11, 12]:

$$\begin{aligned} \frac{1}{2} &< h < 1, \\ \Omega &< 2. \end{aligned} \quad (2.3)$$

Once  $h$  and  $\Omega$  are known, the cosmic scale factor  $R$  is determined through Einstein's field equations. Consequently, the age of the universe  $t_0$  is given in terms of these parameters [13, 9, 14]:

$$t_0 = \frac{9.8 \times 10^9 \text{ y}}{h} \int_0^1 \left( 1 - \Omega + \frac{\Omega}{x^p} \right)^{-1/2} dx. \quad (2.4)$$

$p = 1$  for a matter-dominated (MD) universe, and  $p = 2$  for a radiation-dominated (RD) universe. We will approximate the numerical factor in eq. (2.4) by  $10^{10}$  y. The age of the universe is estimated (independently of the estimates (2.3)) to be [11]:

$$10^{10} \text{ y} < t_0 < 2 \times 10^{10} \text{ y} \quad (2.5)$$

and more probably, in the range  $(1.2-1.8) \times 10^{10}$  y. We define the function  $f(\Omega)$  by rewriting eq. (2.4) as  $t_0 = (f(\Omega)/h) \times 10^{10}$  y. The lower limit in eq. (2.5) puts an additional bound on  $h$  and  $\Omega$ , namely

$$h \leq f(\Omega). \quad (2.6)$$

## 2.2. STABLE NEUTRINOS

For stable primordial neutrinos we require

$$\sum [n(\nu_i)]_0 m(\nu_i) \leq \rho_0, \quad (2.7)$$

where  $[n(\nu_i)]_0$  is the present number density of the neutrino  $\nu_i$ . The sum runs over all neutrino flavors. Neutrinos decouple at a temperature  $T_D$ , when the expansion

rate of the universe becomes larger than their interaction rate:  $T_D \approx 5$  MeV. Neutrinos with a mass  $m(\nu_i) \leq T_D$  decoupled when their number density was equal (up to a statistical factor of order 1) to the photons number density. The present number density of each such flavor of neutrino is

$$[n(\nu_i)]_0 = \frac{3}{11} [n_\gamma]_0 \approx 110/\text{cm}^3. \quad (2.8)$$

Using eqs. (2.1), (2.2) and (2.8), we get from eq. (2.7):

$$\sum m(\nu_i) \leq 100 \Omega h^2 \text{ eV}. \quad (2.9)$$

From eq. (2.6), the weakest limit is derived when  $\Omega[f(\Omega)]^2$  is maximal. For a MD universe, this corresponds to  $\Omega = 2$  and  $h = 0.57$ :

$$\sum m(\nu_i) \leq 65 \text{ eV}. \quad (2.10)$$

For a RD universe, the upper limit corresponds to  $\Omega = 1$  and  $h = \frac{1}{2}$ :

$$\sum m(\nu_i) \leq 25 \text{ eV}. \quad (2.11)$$

The larger the age of the universe, the more stringent these limits are. For example, if  $t_0 = 1.3 \times 10^{10}$  years, the limit is 25 (6.3) eV for a MD (RD) universe. The standard cosmological model assumes that the universe has been MD for most of the time. Thus we use eq. (2.10) as the limit on stable neutrinos mass.

Note that in order to saturate the bound (2.10) we must assume that the universe is closed ( $\Omega > 1$ ) and that all or most of the dark matter consists of neutrinos. If the universe is flat ( $\Omega = 1$ ) or open ( $\Omega < 1$ ) or if particles other than neutrinos have an important contribution to the dark matter, we may obtain much smaller upper bounds.

For neutrinos heavier than a few MeV, eq. (2.8) does not hold: if at the time of decoupling  $T_D < m(\nu_i)$ , the neutrino density  $[n(\nu_i)]_D$  is much smaller than the number density of the photons. If these neutrinos had followed their equilibrium density till the time of decoupling, their number density would have been smaller by the Boltzmann factor  $\exp[-m(\nu_i)/kT_D]$ . However, the small number density suppresses the annihilation rate of the neutrinos long before  $T_D$  is reached. The actual number density obeys [7, 15]

$$\dot{n}(\nu_i) = \langle \sigma_{\text{A}\nu} \rangle \left\{ [n(\nu_i)]^2 - [n_{\text{eq}}(\nu_i)]^2 \right\} - 3Hn(\nu_i), \quad (2.12)$$

where  $\langle \sigma_{\text{A}\nu} \rangle$  is the thermal average of the annihilation rate, and  $n_{\text{eq}}(\nu_i)$  is the number density of the neutrino at equilibrium. This equation is solved numerically. It is found [7, 15] that in the relevant range of mass, the dependence of the neutrino

number density on the neutrino mass can be approximated by  $n(\nu_i) \propto [m(\nu_i)]^{-3}$ . Requiring that the contribution of such neutrinos to the mass density of the universe should not exceed the present upper limit of that density, gives [16]

$$m(\nu_i) \geq 3.4 \text{ GeV} [\Omega h^2]^{-1/2} \quad (2.13)$$

for stable Majorana neutrinos (for Dirac neutrinos the limit is lower by approximately a factor of 2). The weakest limit corresponds to  $\Omega = 2$  and  $h = 0.57$ :

$$m(\nu_i) \geq 4.2 \text{ GeV}. \quad (2.14)$$

Thus, a stable neutrino must be either lighter than 65 eV or heavier than 4.2 GeV.

### 2.3. UNSTABLE NEUTRINOS

Neutrinos with masses between 65 eV and 4.2 GeV may exist, provided that they are unstable: once they decay, the *energy density* of the decay products decreases as  $R^{-4}$ , instead of the  $R^{-3}$  dependence of the *mass density* before the decay.

The contribution of an unstable neutrino to the present energy density of the universe, is related to its mass density at the time of decay  $\tau$ , through

$$\frac{\rho_{\nu 0}}{\rho_{\nu}(\tau)} = \left( \frac{T_0}{T_{\tau}} \right)^4. \quad (2.15)$$

The relation between the neutrino mass density at the time of decay and at the time of decoupling is

$$\frac{\rho_{\nu}(\tau)}{\rho_{\nu}(t_D)} = \frac{4}{11} \left( \frac{T_{\tau}}{T_D} \right)^3. \quad (2.16)$$

Eqs. (2.15) and (2.16) give:

$$\rho_{\nu 0} = \left[ \frac{4}{11} \rho_{\nu}(t_D) \left( \frac{T_0}{T_D} \right)^3 \right] \left( \frac{T_0}{T_{\tau}} \right). \quad (2.17)$$

The quantity in square brackets is the would-be present mass density of the neutrino if it were stable. For a RD universe  $t \propto T^{-2}$ . Thus the limits on unstable neutrinos, analogous to eqs. (2.9) and (2.13) are:

$$\begin{aligned} \sum m(\nu_i) \sqrt{\frac{\tau(\nu_i)}{t_0}} &\leq 100 \Omega h^2 \text{ eV} && \text{for } m(\nu_i) \leq \text{a few MeV}, \\ \sum [m(\nu_i)]^{-2} \sqrt{\frac{\tau(\nu_i)}{t_0}} &\leq 10^{-19} \Omega h^2 \text{ eV}^{-2} && \text{for } m(\nu_i) \geq \text{a few MeV} \end{aligned} \quad (2.18)$$

(where in the second equation we used the approximation  $n(\nu_i) \propto [m(\nu_i)]^{-3}$ ). The weakest limits [14] correspond to  $\Omega = 1$  and  $h = \frac{1}{2}$ :

$$\begin{aligned} [m(\nu_i)]^2 \tau(\nu_i) &\leq 2 \times 10^{20} \text{ eV}^2 \cdot \text{sec} && \text{for } m(\nu_i) \leq \text{a few MeV}, \\ [m(\nu_i)]^{-4} \tau(\nu_i) &\leq 1.5 \times 10^{-22} \text{ eV}^{-4} \cdot \text{sec} && \text{for } m(\nu_i) \geq \text{a few MeV}. \end{aligned} \quad (2.19)$$

Again, if the universe is older than the lower limit in eq. (2.5), or if there are important non-neutrino contributions to the dark matter, the bounds (2.19) become more stringent. For example, if  $t_0 = 1.3 \times 10^{10}$  y, both bounds in eq. (2.19) are lowered by a factor of 12.

If there are charged particles or photons among the decay products of the neutrinos, there are additional astrophysical effects to be considered:

- (i) The black-body radiation background should not be distorted [17, 18].
- (ii) Primordial nucleosynthesis should not be affected [17, 19].
- (iii) The flux of  $\gamma$  rays at the positronium annihilation line should not exceed the observed flux [20].
- (iv) Deuterium should not be destroyed by photodisintegration [21, 22].

Each of these effects leads to an allowed range for the lifetime of such a neutrino. When combined, they give an upper bound of order  $10^4$  sec on the lifetime [23].

If there exist both stable and unstable neutrinos, the bounds on the stable neutrinos masses may be modified: if an unstable neutrino decays into stable neutrinos, their number density increases and the bound on their mass is more stringent; if there are charged particles or photons among the decay products, they affect the temperature of the photon gas but not that of the neutrinos and consequently, the limit on the mass of the stable neutrinos is somewhat relaxed [24].

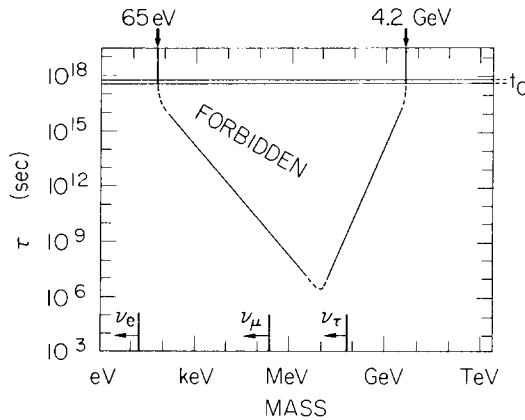


Fig. 1. Cosmological bounds on the mass and the lifetime of neutrinos. The experimental bounds on the known neutrinos masses and the observational bounds on the age of the universe are also shown.



To summarize: *Stable neutrinos are either lighter than 65 eV or heavier than 4.2 GeV. Unstable neutrinos may have a mass in the “forbidden range”, if their lifetime fulfills the bounds (2.19).* The bounds on the neutrino mass and lifetime are shown in fig. 1.

### 3. Neutrino mass-matrices and mixing

#### 3.1. SEE-SAW MATRICES

In the introduction, we discussed how new physics at a scale larger than the  $SU(2)_L$ -breaking scale may explain the lightness of the neutrinos. The best-known realization of this mechanism is the see-saw mass matrix [4].

The general form of the neutrino mass matrix is

$$M = \begin{pmatrix} M_L & m_D \\ m_D^T & M_R \end{pmatrix}. \quad (3.1)$$

In the  $n$  generation case,  $M$  is a  $2n \times 2n$  matrix, and each of the sub-matrices in eq. (3.1) is an  $n \times n$  matrix. We define  $L$ ,  $D$  and  $R$ , the mass scales of  $M_L$ ,  $m_D$  and  $M_R$  respectively:

$$L \equiv [\det M_L]^{1/n}, \quad D \equiv [\det m_D]^{1/n}, \quad R \equiv [\det M_R]^{1/n}. \quad (3.2)$$

$m_D$  is the Dirac mass matrix:

$$(m_D)_{ij} = h_{\phi \nu_L \bar{\nu}_L} \langle \phi \rangle, \quad (3.3)$$

where  $h$  is a Yukawa coupling and  $\phi$  is the usual Higgs doublet of the standard model. As  $\phi$  is needed in order to give mass to other fermions,  $m_D \neq 0$  unless additional symmetries are invoked. In general,  $D$  should be comparable to the charged fermions masses.  $M_R$  and  $M_L$  are the Majorana mass matrices for right-handed and left-handed neutrinos respectively:

$$\begin{aligned} (M_R)_{ij} &= h_{S \bar{\nu}_L \bar{\nu}_L} \langle S \rangle, \\ (M_L)_{ij} &= h_{\Delta_L \nu_L \nu_L} \langle \Delta_L \rangle, \end{aligned} \quad (3.4)$$

where  $S$  is a Higgs singlet and  $\Delta_L$  is a Higgs triplet.

The Weinberg relation,  $M_W = M_Z \cos \theta_W$ , is in good agreement with experiment. In order to preserve it we must have  $\langle \Delta_L \rangle \ll \langle \phi \rangle$ . At the same time,  $\langle S \rangle$  does not break the gauge symmetry of the standard model. Consequently, it may obtain arbitrarily large values. It is likely that  $\langle S \rangle$  is determined by some new physics

beyond the standard model. All such models have scales  $\Lambda \geq \text{TeV}$ , and some even have  $\Lambda \gg \text{TeV}$ . Hence  $R \gg D$ . If  $\phi$ ,  $S$  and  $\Delta_L$  are all present, minimization of the Higgs potential gives [25, 26]  $\langle \Delta_L \rangle \langle S \rangle \sim \langle \phi \rangle^2$ , consistent with the above hierarchy of vev's.

In the following discussion we assume that  $M_L$  is negligible or zero. The neutrino mass matrix of eq. (3.1) acquires the form:

$$M = \begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}. \quad (3.5)$$

Diagonalization of  $M$  gives  $n$  light mass eigenstates, with masses  $O(D^2/R)$ . This is the see-saw principle: the *higher* the scale  $R$ , the *lighter* are these neutrinos. The other  $n$  mass eigenstates are heavy, with masses  $O(R)$ .

$M_R$  may be induced by  $\langle S \rangle$  but it may also be an explicit singlet mass term or a result of a higher order correction in some “beyond standard” theory. We assume that  $R \geq \text{TeV}$ , and give our results in units of  $[R/1 \text{ TeV}]$ . There is no convincing theoretical model with  $R < \text{TeV}$ . However, on purely experimental grounds, the limits (1.1) are consistent with  $R$ -values as low as 50 GeV. Some ad-hoc models may actually assume  $R = 50 \text{ GeV}$ .

The ratio  $D/R$  must be very small. For  $R \geq 1 \text{ TeV}$  and  $D \leq m(\tau)$ , we expect  $D/R \leq 0.002$ . Even if  $R \sim 50 \text{ GeV}$ ,  $D/R \leq 0.04$ .

### 3.2. LEPTON MIXING

Neutrinos with a non-vanishing mass imply possible mixing among generations: the mass eigenstates may differ from the weak interaction states.

The mass matrix (3.5) can be brought into a *block-diagonal* form by a unitary transformation,  $M \rightarrow \mathcal{U}^T M \mathcal{U}$ . We write  $\mathcal{U}$  in terms of four  $n \times n$  submatrices:

$$\mathcal{U} = \begin{pmatrix} \mathcal{U}_a & \mathcal{U}_b \\ \mathcal{U}_c & \mathcal{U}_d \end{pmatrix}, \quad (3.6)$$

which have to fulfill:

$$\mathcal{U}_d^T m_D^T \mathcal{U}_a + \mathcal{U}_b^T m_D^T \mathcal{U}_c + \mathcal{U}_d^T M_R \mathcal{U}_c = 0. \quad (3.7)$$

The light neutrinos mass matrix is:

$$\mathcal{U}_a^T m_D^T \mathcal{U}_c + \mathcal{U}_c^T m_D^T \mathcal{U}_a + \mathcal{U}_c^T M_R \mathcal{U}_c. \quad (3.8)$$

The solution of eq. (3.7), to order  $(D^3/R^2)$  is [27]:

$$\begin{aligned}\mathcal{U}_b &= -\mathcal{U}_c^T = m_D M_R^{-1} + \mathcal{O}\left(\frac{D^3}{R^3}\right), \\ \mathcal{U}_a &= 1 - \frac{1}{2} m_D M_R^{-2} m_D^T + \mathcal{O}\left(\frac{D^4}{R^4}\right), \\ \mathcal{U}_d &= 1 - \frac{1}{2} M_R^{-1} m_D^T m_D M_R^{-1} + \mathcal{O}\left(\frac{D^4}{R^4}\right).\end{aligned}\quad (3.9)$$

The mass matrix for the light neutrinos (eq. (3.8)) is

$$\left[ -m_D M_R^{-1} m_D^T + \mathcal{O}\left(\frac{D^4}{R^3}\right) \right]. \quad (3.10)$$

The mass matrix is brought into a *diagonal* form by an additional unitary transformation,  $\mathcal{U}^T M \mathcal{U} \rightarrow \mathcal{V}^T \mathcal{U}^T M \mathcal{U} \mathcal{V}$ . The matrix  $\mathcal{V}$  is of the form

$$\mathcal{V} = \begin{pmatrix} \mathcal{V}_a & 0 \\ 0 & \mathcal{V}_b \end{pmatrix}. \quad (3.11)$$

Both  $\mathcal{V}_a$  and  $\mathcal{V}_b$  are unitary.

We diagonalized  $M$  in two stages, because in this way the information about the mixing between left-handed and right-handed neutrinos is contained in  $\mathcal{U}$ , while  $\mathcal{V}$  depends on the mixing among generations.

The left-handed neutrinos are related to the mass eigenstates by the following transformation

$$\nu_L = \mathcal{U}_a \mathcal{V}_a \nu_1 + \mathcal{U}_b \mathcal{V}_b \nu_h. \quad (3.12)$$

$\nu_{l(h)}$  are the light (heavy) mass eigenstates. The light neutrinos are almost purely (to  $\mathcal{O}(D/R)$ ) left-handed. The mixing matrix (among the *light* states) for the charged W-boson interactions is  $U = \mathcal{U}_a \mathcal{V}_a$ . As  $\mathcal{U}_a$  is different from the unit matrix only by terms of  $\mathcal{O}(D^2/R^2)$ , we can take as a good approximation  $U = \mathcal{V}_a$ . Thus,  $U$  is unitary to a good approximation. The mixing matrix for the neutral Z-boson interactions is  $\Omega = \mathcal{V}_a^T \mathcal{U}_a^T \mathcal{U}_a \mathcal{V}_a$ , which is somewhat different from the unit matrix because  $\mathcal{U}_a$  is not unitary.

### 3.3. EXPERIMENTAL CONSTRAINTS

As yet, there is no conclusive experimental evidence for non-vanishing lepton mixing. There are two kinds of experiments that put constraints on the mixing terms

in  $U$ :

(i) Depletion experiments, in which the known flux of the neutrino produced in the experiment is compared to the flux of the same flavor of neutrino at some distance.

(ii) Oscillation experiments, in which the known flux of the produced neutrino is compared to the flux of a different flavor at some distance.

These experiments lead to constraints on the mixing angles for sufficiently large  $\Delta m_{ij}^2 = |m_i^2 - m_j^2|$ , but to no limits on these angles for smaller  $\Delta m_{ij}^2$ . For all the range of masses that we are interested in ( $\Delta m_{ij}^2$  larger than tens of  $\text{eV}^2$ ), the asymptotic constraints on  $U$  apply. The strongest upper limits on  $U_{e\mu}$ ,  $U_{e\tau}$  and  $U_{\mu\tau}$  come from the BNL [28] and Fermilab [29] experiments:

$$U_{e\mu} \leq 0.029, \quad U_{e\tau} \leq 0.17, \quad U_{\mu\tau} \leq 0.031. \quad (3.13)$$

These upper bounds were derived under the simplifying assumptions that the mixing matrix is real, and that for each two flavors one may use an effective  $2 \times 2$  mixing matrix.

$U_{e\tau}$  is further limited by  $\pi \rightarrow e\nu$  experiments [30]. The limits are obtained by search for monoenergetic peaks in this decay, and by its branching-ratio measurement. The limits are mass-dependent. Typical values are

$$\begin{aligned} U_{e\tau} &\leq 0.05 && \text{for } m(\nu_\tau) \geq 1 \text{ MeV}, \\ U_{e\tau} &\leq 0.003 && \text{for } m(\nu_\tau) \geq 20 \text{ MeV}. \end{aligned} \quad (3.14)$$

For most of the decay modes that we study, mixing angles as given in eqs. (3.13) and (3.14) are negligible, and we identify the weak doublets with the physical states. We will explicitly use the bounds (3.13) and (3.14) whenever a non-vanishing mixing is significant.

#### 4. Neutrino decays in the extended standard model

The minimal standard model contains neither right-handed neutrinos nor Higgs triplets. Thus, left-handed neutrinos are *exactly* massless. We begin our discussion of neutrino masses by studying an extended standard model, in which right-handed neutrinos are added. In that case, left-handed neutrinos will have masses and may decay. The possible final states for the decay of an unstable neutrino  $\nu_i$  into two or three final particles are:

$$\nu_j + \nu_k + \nu_l, \quad \nu_j + e^+ + e^-, \quad \nu_j + \gamma, \quad \nu_j + \gamma + \gamma, \quad (4.1)$$

where  $\nu_j, \nu_k, \nu_l$  represent any neutrino or antineutrino lighter than  $\nu_i$ . The  $\nu e^+ e^-$

decay mode is allowed only for  $i = \tau$  (as  $m(\nu_i) \geq 2m(e)$  is required). Decays into four or more particles can be safely neglected. We now consider each of these decay modes.

#### 4.1. $\nu_i \rightarrow \nu_j + \nu_k + \bar{\nu}_k$

Unlike other fermions, neutrinos may have flavor changing neutral currents at tree level. A non-vanishing Majorana-mass for right-handed neutrinos breaks down the GIM argument [24], and the Z-boson can mediate a neutrino decay into three lighter ones:  $\nu_i \rightarrow \nu_j \nu_k \bar{\nu}_k$ . As shown in sect. 3, the mixing matrix is  $\Omega = \mathcal{V}_a^T \mathcal{U}_a^T \mathcal{U}_a \mathcal{V}_a$ . Thus, the mixing terms are  $O(D^2/R^2)$ :

$$\Omega_{ij} = [\mathcal{V}_a^T m_D M_R^{-2} m_D^T \mathcal{V}_a]_{ij}. \quad (4.2)$$

We choose to work in a basis where  $m_D$  is diagonal. For example, in the two-generation case

$$m_D = \begin{pmatrix} m_D(\nu_e) & 0 \\ 0 & m_D(\nu_\mu) \end{pmatrix}. \quad (4.3)$$

In such a basis, the charged leptons mass matrix may be non-diagonal, but this does not affect  $\Omega$ . The matrix  $M_R$  is a general symmetric matrix. The matrix  $\mathcal{V}_a$  diagonalizes the light neutrinos mass matrix  $m_D M_R^{-1} m_D^T$ . Typically  $[\mathcal{V}_a]_{ij} \sim m_D(\nu_i)/m_D(\nu_j)$ . Then eq. (4.2) gives:

$$\Omega_{ij} \sim \frac{m_D(\nu_i) m_D(\nu_j)}{R^2}. \quad (4.4)$$

As the neutrino masses are  $m(\nu_i) \sim [m_D(\nu_i)]^2/R$ , we conclude that the coupling of the Z-boson to neutrinos of *different* generations is

$$[\Omega_{ij}]^2 \sim \frac{m(\nu_i) m(\nu_j)}{R^2}. \quad (4.5)$$

The width for the Z-mediated decay into three neutrinos is then estimated to be

$$\Gamma(\nu_i \rightarrow \nu_j \nu_k \bar{\nu}_k) \sim \left[ \frac{m(\nu_i) m(\nu_j)}{R^2} \right] \left[ \frac{m(\nu_i)}{m(\mu)} \right]^5 \Gamma(\mu). \quad (4.6)$$

For the  $\nu_\mu \rightarrow \nu_e \nu_e \bar{\nu}_e$  decay, we assume  $m(\nu_e) \leq 18$  eV and  $R \geq 1$  TeV and obtain

$$\tau(\nu_\mu) [m(\nu_\mu)]^6 \geq 1.6 \times 10^{57} \text{ eV}^6 \text{ sec}. \quad (4.7)$$

This, together with the cosmological bounds (2.19), gives

$$m(\nu_\mu) \geq 80 \text{ MeV} \left[ \frac{R}{1 \text{ TeV}} \right]^{1/5} \left[ \frac{m(\nu_e)}{18 \text{ eV}} \right]^{-1/10}, \quad (4.8)$$

in clear contradiction with the experimental upper bound. We therefore conclude that, if  $\nu_\mu \rightarrow \nu_e \nu_e \bar{\nu}_e$  via Z-exchange were the only decay mode of  $\nu_\mu$ , we must have  $m(\nu_\mu) \leq 65 \text{ eV}$ .

We may now consider the decay  $\nu_\tau \rightarrow \nu_j \nu_k \bar{\nu}_k$ . There are four possible modes, with each of  $j$  and  $k = e$  or  $\mu$ . We assume  $m(\nu_e) \leq 18 \text{ eV}$ ,  $m(\nu_\mu) \leq 65 \text{ eV}$ ,  $R \geq 1 \text{ TeV}$ . We obtain

$$\tau(\nu_\tau) [m(\nu_\tau)]^6 \geq 1.7 \times 10^{56} \text{ eV}^6 \text{ sec}. \quad (4.9)$$

The cosmological bound can be fulfilled with

$$m(\nu_\tau) \geq 65 \text{ MeV} \left[ \frac{R}{1 \text{ TeV}} \right]^{1/5} \left[ \frac{m(\nu_e) + m(\nu_\mu)}{83 \text{ eV}} \right]^{-1/10}. \quad (4.10)$$

Even if we allow  $R$  to be as low as  $50 \text{ GeV}$ , we still get  $m(\nu_\tau) \geq 35 \text{ MeV}$ . *If neutrinos dominantly decay into three lighter neutrinos (via Z-exchange), then  $\nu_\mu$  is lighter than  $65 \text{ eV}$  and  $\nu_\tau$  is either lighter than  $65 \text{ eV}$  or heavier than  $35 \text{ MeV}$ .*

#### 4.2. $\nu_\tau \rightarrow \nu_e e^+ e^-$

If  $\nu_\tau$  is heavier than  $2m(e)$ , it can also decay to a final  $\nu_e e^+ e^-$  state. The dominant contribution is by W-exchange. The decay width can be written as:

$$\Gamma(\nu_\tau \rightarrow \nu_e e^+ e^-) = U_{e\tau}^2 \left[ \frac{m(\nu_\tau)}{m(\tau)} \right]^5 \Gamma(\tau \rightarrow \nu_\tau \bar{\nu}_e e). \quad (4.11)$$

This gives the relation

$$\tau(\nu_\tau) [m(\nu_\tau)]^5 = (U_{e\tau})^{-2} \cdot 3 \times 10^{34} \text{ eV}^5 \text{ sec}. \quad (4.12)$$

The cosmological bound on the energy density (eq. (2.19)) can be fulfilled with  $m(\nu_\tau) \geq 5 \text{ MeV}$ . However, as charged particles are produced in this mode, the additional astrophysical constraints described in sect. 2 apply [22]. Thus we should have  $\tau(\nu_\tau) \leq 10^4 \text{ sec}$ , which can be fulfilled only for  $m(\nu_\tau) \geq 10 \text{ MeV}$ .

#### 4.3. $\nu_i \rightarrow \nu_j + \gamma$

The decay  $\nu_i \rightarrow \nu_j \gamma$  is described by one-loop diagrams, with a W-boson (or a charged Higgs boson) and a charged lepton in the loop. This mode was analyzed in

detail in ref. [31]. The decay width is

$$\Gamma(\nu_i \rightarrow \nu_j \gamma) = \frac{\alpha G_F^2 [m(\nu_i)]^5}{64 \pi^4} \left[ \sum_a U_{ja} U_{ia} F(r_a) \right]^2, \quad (4.13)$$

where  $r_a = [m(\ell_a)/M(W_L)]^2$  and the function  $F(r)$  is given by

$$F(r) = \frac{3}{4(1-r)^2} \left[ -(2-5r+r^2) + \frac{2r^2 \ln r}{1-r} \right]. \quad (4.14)$$

For the three known generations  $r_a \ll 1$ , in which case  $F(r_a) \rightarrow -\frac{3}{2} + \frac{3}{4}r_a$ . From the analysis in sect. 3 we get  $\sum_a U_{ia} U_{ja} = O(D^2/R^2)$ . Thus we expect  $\Gamma \propto r_\tau^2$ , which gives [31]:

$$\tau(\nu_i) [m(\nu_i)]^5 = (U_{i\tau} U_{j\tau})^{-2} 4 \times 10^{43} \text{ eV}^5 \text{ sec}. \quad (4.15)$$

This and the cosmological bound (2.19) give

$$m(\nu_i) \geq \frac{20 \text{ MeV}}{(U_{i\tau} U_{j\tau})^{2/9}}. \quad (4.16)$$

However, the requirement that the radiative lifetime should be shorter than  $10^4$  sec completely excludes this mode.

#### 4.4. $\nu_i \rightarrow \nu_j + \gamma + \gamma$

The  $\nu_i \rightarrow \nu_j \gamma \gamma$  decay [32] proceeds via a box-diagram. Although it is a higher order process than the one-photon decay, it is not GIM suppressed. The dominant contribution comes from the lightest charged lepton which is heavier than the decaying neutrino. Thus, the decay width depends on the  $\mu$  couplings rather than the  $\tau$  couplings. The decay width is

$$\Gamma(\nu_i \rightarrow \nu_j \gamma \gamma) = \frac{\alpha G_F^2 [m(\nu_i)]^5}{64 \pi^4} \left[ \frac{\alpha}{288 \pi} \right] \left[ \frac{m(\nu_i)}{m(\mu)} \right]^4 [U_{j\mu} U_{i\mu}]^2, \quad (4.17)$$

which gives the relation [32]

$$\tau(\nu_i) [m(\nu_i)]^9 = (U_{i\mu} U_{j\mu})^{-2} 6 \times 10^{73} \text{ eV}^9 \text{ sec}. \quad (4.18)$$

This and the cosmological bounds (2.19) give

$$m(\nu_i) \geq \frac{20 \text{ MeV}}{(U_{i\mu} U_{j\mu})^{2/13}}. \quad (4.19)$$

Again, when the astrophysical constraints are taken into account, this mode is excluded.

Thus, *within an extended standard model, only  $\nu_\tau$  can fulfill the cosmological constraints on a heavy neutrino, provided that its mass is between 10 MeV and 70 MeV.*

## 5. Beyond the standard model

### 5.1. NEW DECAY MODES AND DECAY MECHANISMS

In the previous section we concluded that,  $\nu_\mu$  should be lighter than 65 eV and  $\nu_\tau$  is either lighter than 65 eV or with a mass in the range 10–70 MeV. Can these bounds be evaded in models beyond the standard model?

In such models we may have additional neutrino decay modes, corresponding to new light particles suggested by the models. These could be Majorons, familons, sneutrinos and possibly other new particles. In addition, the four modes discussed above (sect. 4) may proceed via new mechanisms. We could have contributions from fourth generation fermions (which have mass  $O(M_W)$  and therefore avoid the GIM-cancellation), right-handed currents, additional “beyond standard” Higgs particles, “horizontal” gauge bosons, unknown effects due to lepton substructure, etc.

In the following sections we discuss these various possibilities. All “beyond standard” theories correspond to a new energy scale  $\Lambda \gg M_W$ . The actual value of  $\Lambda$  may be anywhere between 1 TeV and the Planck mass. It is clear that if  $\Lambda$  is at the GUT scale or at the Planck scale, it is unlikely to lead to fast neutrino decays, and – through the see-saw mechanism – will produce neutrino masses well below 1 eV.

Our best hope for heavier left-handed neutrinos and for faster neutrino decays which could be consistent with the cosmological bounds is from new physics at relatively “nearby” scales around, say, 1 TeV to 1 PeV. Such scales are consistent with left-right symmetric models, horizontal symmetries, and substructure. We will therefore pay special attention to these last cases.

In the remaining of the present section we briefly discuss the case of theories at the GUT scale or the Planck scale. In sects. 6, 7 and 8 we discuss, respectively, the specific cases of left-right symmetry, horizontal symmetry and neutrino substructure, all of which could presumably appear below 1 PeV. In sects. 9 and 10 we discuss the most likely cases of new light particles appearing among the neutrino



decay products: supersymmetric particles and Goldstone bosons (Majorons and familons).

## 5.2. GRAND UNIFIED THEORIES

In the minimal SU(5) model, there are no right-handed neutrinos, and the left-handed neutrinos are massless. However, in other GUTs such as SO(10),  $[SU(3)]^3 \otimes Z_3$  and  $E_6$ , right-handed neutrinos are present. The left-handed neutrinos have very small masses due to the see-saw mechanism. In the SO(10) theory, for example, a Higgs in the 126 representation is needed in order to give the right-handed neutrino a tree-level Majorana mass. *If this Majorana mass is at the scale of the GUT-breaking,  $M_X = O(10^{14} \text{ GeV})$ , the predicted neutrino masses are much smaller than the 65 eV limit on stable neutrinos (eq. (2.10)):*

$$m(\nu_i) \leq \frac{[m(t)]^2}{M_X} \sim 10^{-2} \text{ eV}. \quad (5.1)$$

All light neutrinos are expected to be lighter than the 65 eV limit even for intermediate breaking scales as low as  $10^{11} \text{ GeV}$ . They are certainly light if the new energy scale is the Planck scale  $O(10^{19} \text{ GeV})$ .

In some versions of GUT, Higgs representations needed to give right-handed neutrinos Majorana masses at tree-level are absent.  $M_R \neq 0$  can still be a result of loop-diagrams. In such a case, there are several interesting consequences:

(i) The scale of the right-handed neutrino mass,  $R$ , is smaller than the GUT breaking scale,  $M_X$ . For example, in the minimal SO(10) model,  $M_R$  arises at the two-loop level [33]:

$$R = \left[ \frac{\alpha}{\pi} \right]^2 \left[ \frac{D}{M(W_L)} \right] \epsilon M_X, \quad (5.2)$$

where  $\epsilon$  is a mixing factor. Consequently, the light-neutrino mass may be larger than 65 eV:

$$m(\nu_i) \sim \left[ \epsilon \left( \frac{\alpha}{\pi} \right)^2 \right]^{-1} \left[ \frac{M(W_L)}{m(u_i)} \right] \left( \frac{[m(u_i)]^2}{M_X} \right), \quad (5.3)$$

which gives (for  $\epsilon = 0.1$  and  $m(t) = 45 \text{ GeV}$ ):

$$m(\nu_\mu) \sim 2 \text{ keV}, \quad m(\nu_\tau) \sim 70 \text{ keV}. \quad (5.4)$$

In other such models (e.g. minimal  $E_6$ )  $M_R$  arises at the one-loop level, and thus  $m(\nu_i)$  are much smaller.

(ii) The  $M_R$  matrix may have some hierarchy, which depends on the hierarchy in  $m_D$ . In the minimal SO(10) model [33]

$$M_R \propto m_D \rightarrow \frac{m(\nu_i)}{m(\nu_j)} \sim \frac{m(u_i)}{m(u_j)}. \quad (5.5)$$

The implications of such a relation are discussed in sect. 11.

(iii) The neutrino Dirac masses may be related to different fermionic sectors. As implied in the former equation, in the SO(10) model  $m_D(\nu_i) \sim m_D(u_i)$  at the GUT scale ( $u_i$  are the up-sector quarks).

We conclude that in most grand unified theories or gravity-related models, neutrino masses are inversely proportional to  $M_X$  and are well below 1 eV. In some rare cases, the theory may allow neutrino masses above 65 eV. However, in these cases, the decay mechanisms of these neutrinos will be either the ones discussed in sect. 4 or additional mechanisms, in which the decay amplitude is inversely proportional to powers of  $M_X$  and the resulting lifetime cannot obey the cosmological constraints. *Hence, the conclusions stated at the end of sect. 4 remain valid in all models with Majorana masses at energy scales above  $O(10^{11})$  GeV including all GUTs.* The only way around these conclusions are GUTs which contain LRS or a horizontal symmetry at a lower energy scale. We discuss these two cases in the next two sections.

## 6. Neutrino decay in left-right symmetric models

### 6.1. THE MODEL

In LRS models, the electroweak group is extended to an  $SU(2)_L \otimes SU(2)_R \otimes U(1)_{B-L}$  gauge group. Leptons transform as  $(\frac{1}{2}, 0)_{-1} + (0, \frac{1}{2})_{-1}$  representations. In the minimal LRS model [25], the Higgs fields  $\Phi$ ,  $\Delta_L$  and  $\Delta_R$  transform like  $(\frac{1}{2}, \frac{1}{2})_0$ ,  $(1, 0)_2$  and  $(0, 1)_2$  representations, respectively. The model is “minimal” in the sense that it has the minimal Higgs content that gives hierarchical symmetry breaking and predicts heavy right-handed neutrinos and very light left-handed ones.

The neutral components of the  $\Phi$  field,  $\phi_1^0$  and  $\phi_2^0$ , have vev's  $k_1$  and  $k_2$  respectively, while those of  $\Delta_L$  and  $\Delta_R$  get vev's  $v_L$  and  $v_R$  respectively. The neutrino mass matrix is then:

$$M = \begin{pmatrix} -hv_L & \frac{1}{2}(h_1 k_1 + h_2 k_2) \\ \frac{1}{2}(h_1^T k_1 + h_2^T k_2) & hv_R \end{pmatrix}. \quad (6.1)$$

$h_1$  and  $h_2$  are the Yukawa coupling matrices (in generation space) of  $\phi_1$  and  $\phi_2$  (defined in eq. (3.3)), while  $h$  is the Yukawa coupling matrix for  $\Delta_L$  and  $\Delta_R$  (note

that the two couplings defined in eq. (3.4) are equal due to parity). The charged leptons masses are  $\frac{1}{2}(h_2 k_1 + h_1 k_2)$ .

The vev's fulfill [25]

$$v_L v_R = O(k^2), \quad (6.2)$$

where  $k^2 \equiv k_1^2 + k_2^2$  (see appendix A for a detailed analysis). This implies that one may consistently assume both  $v_R^2 \gg k^2$  (necessary for  $W_R$  to be heavier than  $W_L$ ) and  $k^2 \gg v_L^2$  (necessary for  $M(W_L) = M(Z)\cos\theta_w$ ). Thus we take

$$v_R^2 \gg k^2 \gg v_L^2. \quad (6.3)$$

Consequently, the matrix (6.1) is a see-saw matrix, as discussed in sect. 3.

## 6.2. $\nu_i \rightarrow \bar{\nu}_j \nu_k \nu_l$

The  $\Delta_L$  Higgs field has Yukawa couplings to leptons [25]:

$$\sum_{a,b=e,\mu,\tau} h_{ab} \Psi_{La}^T C i \tau_2 \Delta_L \Psi_{Lb}, \quad (6.4)$$

where  $\Psi_L$  is the left-handed leptonic doublet,  $\Psi_L = \begin{bmatrix} \nu_L \\ e_L \end{bmatrix}$ , and  $C$  is the charge-conjugation matrix. Thus  $\Delta_L^0$  mediates the decay [34]  $\nu_i \rightarrow \bar{\nu}_j \nu_k \nu_l$  (see fig. 2a) with an amplitude proportional to

$$\frac{h_{ij} h_{kl}}{[M(\Delta_L^0)]^2}. \quad (6.5)$$

In the  $\nu_\mu$  case, the only possible mode is  $\nu_\mu \rightarrow \bar{\nu}_e \nu_e \nu_e$ . The width of this decay is proportional to

$$\frac{(h_{e\mu} h_{ee})^2}{[M(\Delta_L^0)]^4} [m(\nu_\mu)]^5. \quad (6.6)$$

We do not know the values of  $M(\Delta_L^0)$ ,  $h_{ee}$  and  $h_{e\mu}$ . Consequently, we cannot derive a relation between  $m(\nu_\mu)$  and  $\tau(\nu_\mu)$ . However [14], the  $\Delta_L^{++}$  member of the  $\Delta_L$

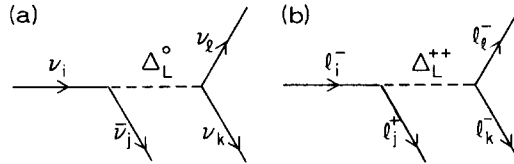


Fig. 2.  $\Delta_L$  exchange decays in LRS models: (a)  $\nu_i \rightarrow \bar{\nu}_j \nu_k \nu_l$ , (b)  $\ell_i^- \rightarrow \ell_j^+ \ell_k^- \ell_l^-$ .

Higgs triplet can mediate the decay  $\mu^- \rightarrow e^+ e^- e^-$  (fig. 2b). The amplitude for this decay is related to the  $\nu_\mu$  decay amplitude through the gauge symmetry. *The Yukawa couplings are exactly the same as in eq. (6.6).* The decay width is thus proportional to

$$\frac{(h_{e\mu}h_{ee})^2}{[M(\Delta_L^{++})]^4} [m(\mu)]^5. \quad (6.7)$$

All other factors are equal for both widths. The ratio between the widths is [14]

$$\frac{\Gamma(\nu_\mu \rightarrow \bar{\nu}_e \nu_e \nu_e)}{\Gamma(\mu^- \rightarrow e^+ e^- e^-)} = \left[ \frac{M(\Delta_L^{++})}{M(\Delta_L^0)} \right]^4 \left[ \frac{m(\nu_\mu)}{m(\mu)} \right]^5. \quad (6.8)$$

The three components of the  $\Delta_L$  triplet are approximately degenerate, with mass-squared  $O(v_R^2)$  (see appendix A for a detailed study):

$$\frac{|[M(\Delta_L^{++})]^2 - [M(\Delta_L^0)]^2|}{[M(\Delta_L^{++})]^2} \sim O\left[\frac{k^2}{v_R^2}\right] \sim O\left[\frac{M(W_L)}{M(W_R)}\right]^2 < 2.5 \times 10^{-3}. \quad (6.9)$$

As  $[M(\Delta_L^{++})/M(\Delta_L^0)]^4 = 1 + O(10^{-3})$ , eq. (6.8) reduces to [14]:

$$\frac{\Gamma(\nu_\mu \rightarrow \bar{\nu}_e \nu_e \nu_e)}{\Gamma(\mu^- \rightarrow e^+ e^- e^-)} = \left[ \frac{m(\nu_\mu)}{m(\mu)} \right]^5. \quad (6.10)$$

The mass and the lifetime of the  $\mu$ -lepton are

$$m(\mu) = 105.7 \text{ MeV}, \quad \tau(\mu) = 2.2 \times 10^{-6} \text{ sec} \quad (6.11)$$

and the experimental upper bound on the branching ratio is [35]

$$\text{BR}(\mu \rightarrow 3e) \leq 2.4 \times 10^{-12}. \quad (6.12)$$

Then eq. (6.10) gives

$$\tau(\nu_\mu) [m(\nu_\mu)]^5 \geq 1.2 \times 10^{46} \text{ eV}^5 \cdot \text{sec}. \quad (6.13)$$

Combining this with the cosmological bound (2.19) one obtains

$$m(\nu_\mu) \geq 35 \text{ MeV}, \quad (6.14)$$

in clear conflict with the experimental bound  $m(\nu_\mu) \leq 250 \text{ keV}$ . *Within LRS models  $\nu_\mu$  cannot be heavier than 65 eV.*

Can such a model accommodate a  $\nu_\tau$ , with  $m(\nu_\tau)$  anywhere between 65 eV and 70 MeV? We now analyze the  $\Delta_L$ -mediated  $\nu_\tau$  decay.

In general, the  $\Delta_L^0$  exchange provides  $\nu_\tau$  with six different decay modes:

$$\nu_\tau \rightarrow \bar{\nu}_\mu \nu_\mu \nu_\mu, \bar{\nu}_\mu \nu_\mu \nu_e, \bar{\nu}_\mu \nu_e \nu_e, \bar{\nu}_e \nu_\mu \nu_\mu, \bar{\nu}_e \nu_\mu \nu_e, \bar{\nu}_e \nu_e \nu_e. \quad (6.15)$$

The decay width, summed over all six modes, is proportional to

$$\Gamma(\nu_\tau \rightarrow \bar{\nu}_i \nu_j \nu_k) \propto \frac{\sum_{ijk} (h_{\tau i} h_{jk})^2}{[M(\Delta_L^0)]^4} [m(\nu_\tau)]^5. \quad (6.16)$$

However, there are also six possible decay modes for the  $\tau$  lepton, mediated by the  $\Delta_L^{++}$  Higgs particle:

$$\tau \rightarrow \mu^+ \mu^- \mu^-, \mu^+ \mu^- e^-, \mu^+ e^- e^-, e^+ \mu^- \mu^-, e^+ \mu^- e^-, e^+ e^- e^-. \quad (6.17)$$

The total decay width for these modes is proportional to

$$\Gamma(\tau^- \rightarrow \ell_i^+ \ell_j^- \ell_k^-) \propto \frac{\sum_{ijk} (h_{\tau i} h_{jk})^2}{[M(\Delta_L^{++})]^4} [m(\tau)]^5. \quad (6.18)$$

The combinations of Yukawa couplings which appear in eqs. (6.16) and (6.18) are identical. The same argument as in the case of  $\nu_\mu$  decay now yields:

$$\frac{\Gamma(\nu_\tau \rightarrow \bar{\nu}_i \nu_j \nu_k)}{\Gamma(\tau^- \rightarrow \ell_i^+ \ell_j^- \ell_k^-)} = \left[ \frac{m(\nu_\tau)}{m(\tau)} \right]^5. \quad (6.19)$$

The mass and the lifetime of the  $\tau$  lepton are

$$m(\tau) = 1784 \text{ MeV}, \quad \tau(\tau) = 2.9 \times 10^{-13} \text{ sec}. \quad (6.20)$$

The ARGUS collaboration has recently reported a new experimental upper bound for all channels of  $\tau \rightarrow 3\ell$ . They obtain [36]:

$$\text{BR}(\tau \rightarrow 3\ell) \leq 3.8 \times 10^{-5}. \quad (6.21)$$

Then eq. (6.19) gives

$$\tau(\nu_\tau) [m(\nu_\tau)]^5 \geq 1.4 \times 10^{38} \text{ eV}^5 \cdot \text{sec}. \quad (6.22)$$

Combining this with the cosmological bound (2.19) we obtain:

$$m(\nu_\tau) \geq 900 \text{ keV}. \quad (6.23)$$

We conclude: *Within LRS models, in order to obey the cosmological bound on the*

neutrino mass and lifetime,  $m(\nu_\tau)$  must be either below 65 eV or between 0.9 MeV and 70 MeV.

### 6.3. $\nu_\tau \rightarrow e^+ e^- \nu_j$

The  $\Delta_L^+$  particle may mediate the decay  $\nu_\tau \rightarrow e^+ e^- \nu_j$ . This process is related to the  $\Delta_L^0$ -mediated decay discussed in this section. However, there are three differences between the decay rates:

(i) The phase-space factor may be important for the  $\nu e^+ e^-$  final state. This process has a threshold energy of 1 MeV.

(ii) While there are six possible final states for  $\bar{\nu}_j \nu_k \nu_l$  (eq. (6.15)), there are only two for the  $e^+ e^- \nu_j$  final state, namely  $j = e$  or  $\mu$ .

(iii) The Yukawa couplings are different (the relations are given by Clebsch-Gordan coefficients).

All of these differences lead to:

$$\Gamma(\nu_\tau \rightarrow e^+ e^- \nu_j) \leq \Gamma(\nu_\tau \rightarrow \bar{\nu}_j \nu_k \nu_l). \quad (6.24)$$

As discussed in sect. 2, when there are charged particles among the decay products, additional astrophysical bounds apply. The strongest of these comes from deuterium photodisintegration. We have followed the calculations by Lindley in ref. [21]. We conclude that one obtains a strong limit on the  $\nu_\tau$  lifetime (of order  $10^4$  sec) for  $\nu_\tau$  heavier than about 10 MeV and a non-negligible  $\text{BR}(\nu_\tau \rightarrow e^+ e^- \nu)$  (e.g.  $\geq 10^{-4}$ ). However, no range of  $\nu_\tau$  mass is excluded by this effect. Consequently, our conclusions do not change when taking into account this decay mode and, in particular, a  $\nu_\tau$  mass above 0.9 MeV is allowed.

### 6.4. $\nu_i \rightarrow \nu_j \gamma$ [37]

Left-right mixing may enhance the radiative neutrino decay, due to the lack of GIM cancellation. However, the decay rate depends on the mixing between  $W_L$  and  $W_R$ . The smaller the mixing, the smaller is the enhancement. Thus, the effect is not large enough to allow a fast radiative decay, so as to avoid the cosmological constraints. There is no enhancement of the two photon decay.

We conclude: *in the minimal LRS model, the masses of  $\nu_\mu$  and  $\nu_\tau$  are constrained to be in the range:*

$$\begin{aligned} m(\nu_\mu) &\leq 65 \text{ eV}, \\ m(\nu_\tau) &\leq 65 \text{ eV} \quad \text{or} \quad 0.9 \text{ MeV} \leq m(\nu_\tau) \leq 70 \text{ MeV}. \end{aligned} \quad (6.25)$$

## 7. Neutrino decay with generation-changing gauge bosons

In order to explain the “generations puzzle”, different “horizontal” symmetries were suggested. The horizontal group may be *discrete* – in which case it does not lead to additional mechanisms for neutrino decays – or *continuous*. If it is continuous, it can be global or local. The case of a *global* symmetry is discussed in sect. 10. Here we discuss the implications of a horizontal *gauge* group, H.

There are severe limitations on the group H, coming from both the requirement of theoretical consistency and phenomenology. These limitations make it difficult to construct a completely satisfactory model. There is also the possibility of a horizontal gauge symmetry in which generation-changing interactions can be “diagonalized away”.

For our purposes, the only case we must consider is a model with a gauge group  $G \otimes H$  where G acts within a generation and fulfills  $G \supseteq SU(2)_L \otimes U(1)_Y$ , and the structure of H and its breaking are such that *flavor-changing gauge interactions* are induced.

The leptonic interaction states are eigenstates of the diagonal generators of both G and H. *As G and H commute,  $SU(2)_L$  partners have the same quantum numbers under H.*

The mass eigenstates are related to the interaction eigenstates by the following transformations:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \mathcal{U}^N \begin{pmatrix} \nu_e^i \\ \nu_\mu^i \\ \nu_\tau^i \end{pmatrix}, \quad \begin{pmatrix} e \\ \mu \\ \tau \end{pmatrix} = \mathcal{U}^L \begin{pmatrix} e^i \\ \mu^i \\ \tau^i \end{pmatrix}. \quad (7.1)$$

As explained in sect. 3,  $\mathcal{U}^N$  is approximately unitary ( $\mathcal{U}^L$  is exactly unitary). Moreover, *as the leptonic mixing angles are small,  $\mathcal{U}^N \approx \mathcal{U}^L$ .*

The coupling of the horizontal gauge boson  $H^a$  to the neutrinos  $\nu_i \bar{\nu}_j$  is  $g_H \mathcal{U}_{ip} T_{pq}^a \mathcal{U}_{jq}^*$ , where  $g_H$  is the gauge coupling and  $T$  is a generator of the group H. Again, we do not know either the mass of  $H^a$  or its gauge couplings and we cannot calculate the decay rate. However,  $H^a$  has exactly the same coupling to the charged leptons  $\ell_i^- \ell_j^+$ . Consequently, *the amplitude for the  $H^a$ -mediated decay  $\nu_i \rightarrow \nu_j \nu_k \bar{\nu}_l$  (fig. 3a) is exactly equal to the amplitude for the decay  $\ell_i^- \rightarrow \ell_j^- \ell_k^- \ell_l^+$  (fig. 3b): all couplings, mixing angles and the intermediate boson mass are the same for both amplitudes. Thus, the ratio between the decay widths is just the ratio between the phase-space factors.*

Both  $\nu_\mu$  and  $\mu$  have one such possible mode, with the relation:

$$\frac{\Gamma(\nu_\mu \rightarrow \nu_e \nu_e \bar{\nu}_e)}{\Gamma(\mu \rightarrow e^- e^- e^+)} = \left[ \frac{m(\nu_\mu)}{m(\mu)} \right]^5. \quad (7.2)$$

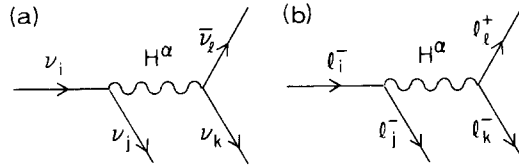


Fig. 3.  $H^a$  exchange decays in models with a horizontal gauge symmetry: (a)  $\nu_i \rightarrow \nu_j \nu_k \bar{\nu}_l$ , (b)  $\ell_i^- \rightarrow \ell_j^- \ell_k^- \ell_l^+$ .

This is exactly the result (6.10), and thus the same conclusions follow: *Within horizontal models, where the horizontal gauge group commutes with the standard model gauge group,  $\nu_\mu$  should be lighter than 65 eV.*

As for  $\nu_\tau$  decay, the  $H^a$  exchange may provide  $\nu_\tau$  with the same six decay modes of the LRS model (eq. (6.15)) (two of these modes,  $\bar{\nu}_\mu \nu_\mu \nu_e$  and  $\bar{\nu}_e \nu_\mu \nu_e$ , have now each two possible diagrams. This does not change any of our results). At the same time, it provides  $\tau$  with six possible decay modes (eq. (6.17)). For each  $\nu_\tau$ -decay amplitude there is a corresponding, exactly equal, amplitude for  $\tau$  decay. The sum of the squared amplitudes is exactly equal, and we get:

$$\frac{\Gamma(\nu_\tau \rightarrow 3\nu)}{\Gamma(\tau \rightarrow 3\ell)} = \left[ \frac{m(\nu_\tau)}{m(\tau)} \right]^5. \quad (7.3)$$

The conclusions of eq. (6.19) follow again: *Within models with horizontal gauge symmetry,  $m(\nu_\tau)$  must be either below 65 eV or between 0.9 and 70 MeV.*

The analysis of the  $\nu_\tau \rightarrow e^+ e^- \nu_j$  mode is similar to the LRS case, with the same conclusions. Once the algebra  $H$  and the leptonic representations are specified, all the couplings are known, and the branching ratio for this decay is determined.

The radiative decays cannot be mediated by the horizontal gauge bosons which are electrically neutral.

These bounds on the masses of  $\nu_\mu$  and  $\nu_\tau$  apply to a much broader class of models: in any model where the dominant decay mode is the  $3\nu$  decay through an  $SU(2)_L$ -singlet exchange, the bounds obtained here are valid. The same particle couples to both neutrinos and charged leptons which are their  $SU(2)_L$  partners, and with equal strength. Thus the bounds (6.13) and (6.22) hold, leading together with the cosmological bound to

$$m(\nu_\mu) \leq 65 \text{ eV},$$

$$m(\nu_\tau) \leq 65 \text{ eV} \quad \text{or} \quad 0.9 \text{ MeV} \leq m(\nu_\tau) \leq 70 \text{ MeV}. \quad (7.4)$$

Simple group theoretical considerations show that these limits also apply to the case of a Higgs triplet with  $B - L = 0$ , namely: the same Higgs particle couples to



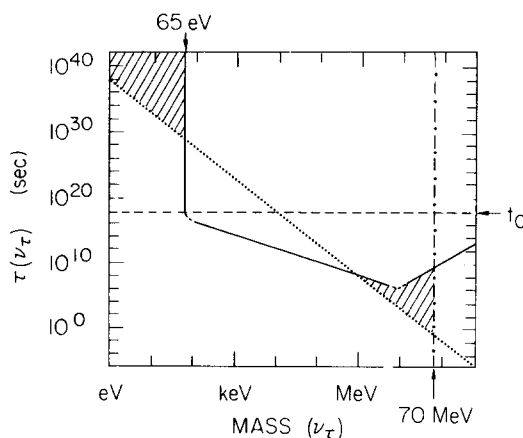


Fig. 4. The mass and the lifetime of  $\nu_\tau$ . The shaded regions are those allowed in the models of sects. 4–7. The solid line gives the cosmological constraints, the dotted line gives the particle-physics constraints and the dashed-dotted line gives the direct experimental limit. The age of the universe is shown in dashed line.

neutrinos and their  $SU(2)_L$  partners, and with equal strength. Consequently the same results apply.

The allowed range for the mass and the lifetime of  $\nu_\tau$ , if it decayed via the channels discussed so far, is shown in fig. 4.

## 8. Substructure models

Another possible future direction for physics beyond the standard model is the hypothesis of quark and lepton substructure, possibly accompanied by a substructure of Higgs particles, W and Z. At present, there are no convincing explicit substructure models. However, we can describe possible low energy effects of such schemes in terms of effective interaction terms. In particular, if neutrinos have substructure at a typical energy scale  $\Lambda$ , we expect effective terms like

$$\frac{g^2}{\Lambda^2} \bar{\nu}_i \nu_j \bar{\nu}_k \nu_l \quad (8.1)$$

where  $\nu$  is a left-handed or a right-handed neutrino or antineutrino and  $g$  may be an effective strong coupling constant. Such terms could contribute to decays like  $\nu_i \rightarrow \nu_j \nu_k \nu_l$  and analogous terms could induce other neutrino decays.

The present *model-independent* bounds of  $\Lambda$  for all leptons and quarks are around  $\Lambda \sim O(\text{TeV})$ . However, *model-dependent* bounds involving *generation-changing* transitions (e.g.  $\mu \rightarrow e\gamma$ ,  $\mu \rightarrow 3e$ ,  $K \rightarrow e\mu$ ,  $\Delta M(K_S^0 - K_L^0)$ ,  $K \rightarrow \pi e\mu$ ) lead to higher values of  $\Lambda$ , typically between 100 TeV and a few PeV [38].

Since all neutrino decays involve generation-changing transitions, we must use the latter range of  $\Lambda$ -values. We obtain, e.g.

$$\Gamma(\nu_i \rightarrow \nu_j \nu_k \nu_l) \sim \frac{[m(\nu_i)]^5}{\Lambda^4}, \quad (8.2)$$

or

$$\tau(\nu_i)[m(\nu_i)]^5 \sim \left[ \frac{\Lambda}{100 \text{ TeV}} \right]^4 \cdot 7 \times 10^{40} \text{ eV}^5 \text{ sec}. \quad (8.3)$$

This, together with the cosmological bounds, gives

$$m(\nu_i) \geq 9 \text{ MeV} \left[ \frac{\Lambda}{100 \text{ TeV}} \right]^{4/9}. \quad (8.4)$$

If neutrinos have substructure, and the dominant contribution to their decay comes from amplitudes with the characteristic substructure scale, then  $\nu_\mu$  is lighter than 65 eV and  $\nu_\tau$  is either lighter than 65 eV or between 9 and 70 MeV.

## 9. Supersymmetry

Supersymmetry introduces many additional particles. These particles, if lighter than the neutrino, may allow additional final states for neutrino decays. For each decay mode discussed in the previous sections there is a corresponding mode with two of the final particles replaced by their super-partners, provided that the final particles are *lighter* than the decaying neutrino.

### 9.1. $\nu_i \rightarrow \nu_j \tilde{\nu} \tilde{\nu}$

Of particular interest is the scalar partner of the neutrino, the sneutrino  $\tilde{\nu}$ . There are no *model-independent* bounds on the sneutrino masses. However, there are *model-dependent* lower bounds:

(i) The apparent absence of the decays  $\ell_i \rightarrow \ell_j \tilde{\nu}_i \tilde{\nu}_j$  implies a lower bound on the sum of the masses of any two different sneutrinos [39]:

$$\begin{aligned} m(\tilde{\nu}_\tau) + m(\tilde{\nu}_{e,\mu}) &\geq m(\tau), \\ m(\tilde{\nu}_\mu) + m(\tilde{\nu}_e) &\geq m(\mu). \end{aligned} \quad (9.1)$$

This bound holds if the mass of the wino  $M_{\tilde{W}}$ , is not much larger than  $M_W$ .

(ii) The process  $e^+e^- \rightarrow \gamma \tilde{\nu} \tilde{\nu}$  should enhance the single-photon production in  $e^+e^-$  annihilation. If  $M_{\tilde{W}} \leq 60 \text{ GeV}$ , one gets lower bounds on sneutrino masses (e.g.  $m(\tilde{\nu}) \geq 10 \text{ GeV}$  for  $M_{\tilde{W}} \sim 30 \text{ GeV}$ ) [40].

(iii) If the difference between the masses of up-squarks and sneutrinos is large, universality may be violated [41]. The bounds depend on the masses of the wino and the photino.

Thus, if  $M_{\tilde{W}} \sim M_W$ , there is at most one extremely light sneutrino. We label such a hypothetical sneutrino by  $\tilde{\nu}_0$ .

We do not know any reasonable model which predicts an extremely light sneutrino. In the limit of exact SUSY, sneutrino squared masses are  $O(D^4/R^2)$ . When local SUSY is broken in a hidden sector, all the sneutrinos acquire a common mass-squared which is expected to be at least tens GeV. At tree level we expect the sneutrinos to be heavy compared to the neutrinos, and almost degenerate. Even if radiative corrections are significant, an extreme fine-tuning is needed to render the sneutrino practically massless. This is very unlikely, but we cannot rigorously rule it out.

Assuming the existence of a sneutrino  $\tilde{\nu}_0$  lighter than the neutrinos, a neutrino may decay via  $\nu_i \rightarrow \nu_j \tilde{\nu}_0 \tilde{\nu}_0$ . This mode may proceed via the following mechanisms:

(i) Z-boson exchange. This is suppressed by the smallness of the neutral mixings, as discussed in sect. 4.

(ii) Zino exchange. The amplitude is proportional to the mixing between  $\tilde{\nu}_0$  and a neutrino of a different generation.

(iii) Exchange of particles that arise in models beyond the supersymmetric standard model, e.g. a  $\Delta_L$  Higgs triplet, a horizontal gauge boson, or their supersymmetric partners. In these cases, the amplitude is inversely proportional to powers of the mass of the intermediate particle. *The  $SU(2)_L$ -related process,  $\ell_i \rightarrow \ell_j \tilde{\ell}_0 \tilde{\ell}_0$ , is forbidden because the charged leptons are experimentally known to be lighter than their spartners.*

If the decays of  $\nu_\mu$  and/or  $\nu_\tau$  proceed via the mechanisms (ii) or (iii), their lifetime may be short enough to evade the cosmological bounds. In such a scenario both  $\nu_\mu$  and  $\nu_\tau$  may have masses in all the experimentally allowed range. However, this requires the existence of one extremely light sneutrino, an unlikely possibility.

## 9.2. OTHER DECAY MODES

Charged sleptons have a lower bound on their masses, of order 20 GeV. Consequently, the mode  $\nu_\tau \rightarrow e \tilde{e} \tilde{\nu}$  is kinematically forbidden.

The photino may be massless. In this case, quark-lepton universality together with the lower bounds on squark masses ( $\geq 60$  GeV) forbid an extremely light sneutrino in a large class of models [41]. Consequently,  $\nu_i \rightarrow \tilde{\gamma} \tilde{\nu}$  is forbidden.

The decay mode  $\nu_i \rightarrow \tilde{\gamma} \tilde{\nu}_j$  is not excluded. However, it is expected to be suppressed by powers of  $[m(\mu)/m(\tilde{\ell})]$  compared to  $\nu_i \rightarrow \gamma \gamma \nu_j$ , and therefore is not relevant to our discussion.

We conclude: *Present experimental bounds on sparticles masses allow additional neutrino decay modes which may, in principle, fulfill the cosmological bounds for any*

neutrino mass. However, the sparticle mass spectrum needed for this scenario does not appear to arise in any reasonable known model.

## 10. Spontaneously broken global symmetries

When a *global* symmetry is spontaneously broken, *massless Goldstone bosons* appear. This may suggest new decay modes:  $\nu_i \rightarrow \nu_j + \text{Goldstone boson}$ .

The decay rates depend on the symmetry breaking scale. If the Goldstone boson interacts with charged fermions, there are bounds on this scale. These bounds come from

- (i) astrophysical considerations of stellar energy loss [42,43];
- (ii) experimental bounds on charged lepton decay through a Goldstone boson emission [44].

There are no limits on the interaction scale of Goldstone bosons that interact with neutrinos only.

There are several models with global symmetry which are relevant to neutrino decays:

- (i) The triplet majoron model [45]. Here, neutrinos are light as a result of new physics *at a scale much smaller than the weak interaction scale*. However, as explained in the introduction, we believe that such a possibility is unnatural. We assume that the extreme lightness of neutrinos is a result of new physics at a high energy scale. Thus, we do not discuss this possibility any further.
- (ii) The singlet majoron model [10].
- (iii) The familon model [46].
- (iv) Other, more “exotic” models, which we briefly mention in the end of this section.

### 10.1. LEPTON-NUMBER SYMMETRY

The minimal standard model has a global lepton-number symmetry. Once right-handed neutrinos are introduced, terms of the form  $\nu_R \nu_R$  may appear (due to  $\nu_R$  being a gauge singlet). This leads to the breaking of the lepton-number symmetry. This breaking can be spontaneous, if the standard model is further extended to include an  $SU(2)_L \otimes U(1)_Y$  Higgs singlet  $S$  that carries  $L = -2$ . When  $S$  gets a vev:

- (i) The right-handed neutrinos get a Majorana mass term,  $h\langle S \rangle \nu_R \nu_R$ . Here  $h$  is a Yukawa coupling. The mass scale of  $M_R$  is  $R = h\langle S \rangle$ .
  - (ii) A massless Goldstone boson appears, the majoron  $J = \text{Im } S$ .
- In sect. 3 we showed that the light neutrino mass matrix is (eq. (3.8))

$$\mathcal{U}_a^T m_D \mathcal{U}_c + \mathcal{U}_c^T m_D^T \mathcal{U}_a + \mathcal{U}_c^T M_R \mathcal{U}_c. \quad (10.1)$$

The majoron couples to *right-handed* neutrinos only. As  $J = \text{Im } S$ , its coupling to the right-handed neutrinos is  $M_R / \langle S \rangle$ . As the *light* neutrinos have a small right-handed component, the majoron couples to them. The Yukawa coupling

matrix in the light sector is derived by the  $\mathcal{U}$ -rotation:

$$\frac{1}{\langle S \rangle} \mathcal{U}_c^T M_R \mathcal{U}_c. \quad (10.2)$$

As the matrices (10.1) and (10.2) cannot, in general, be simultaneously diagonalized, the majoron has non-diagonal couplings in the light neutrino sector. These couplings allow the decay [10]

$$\nu_i \rightarrow J \bar{\nu}_j. \quad (10.3)$$

When taking  $\mathcal{U}_a$  and  $\mathcal{U}_c$  as given in eq. (3.9), one finds that the mass matrix for the light neutrinos (10.1) is  $[-m_D M_R^{-1} m_D^T + O(D^4/R^3)]$ , while the Yukawa coupling matrix (10.2) is  $(1/\langle S \rangle)[m_D M_R^{-1} m_D^T + O(D^4/R^3)]$ . Consequently, the Yukawa coupling matrix and the mass matrix can be simultaneously diagonalized up to  $O(D^3/R^3)$ . *The leading non-diagonal  $J \nu_e \nu_\mu$  Yukawa coupling is  $O(D^4/R^4)$ .* This is the result obtained in ref. [27]. *However, the majoron does not have any coupling, diagonal or non-diagonal, to a massless neutrino. The decay (10.3) is not allowed if the decay product  $\nu_j$  is exactly massless.* This is because the majoron couples to a light neutrino only if this neutrino has a right-handed component. An exactly massless neutrino, however, is purely left-handed. Consequently, it is decoupled from the majoron.

The most likely situation is that  $\nu_j$  is much lighter than  $\nu_i$ , but not massless. Then the amplitude for the decay  $\nu_i \rightarrow J \bar{\nu}_j$  is proportional to  $m_D(\nu_j)[m_D(\nu_i)]^3/R^4$ . The decay width is estimated

$$\Gamma(\nu_i \rightarrow J \bar{\nu}_j) = \frac{h^2}{16\pi} \left[ \frac{[m_D(\nu_i)]^6 [m_D(\nu_j)]^2}{R^8} \right] m(\nu_i). \quad (10.4)$$

The masses of the light neutrinos are  $m(\nu_k) \sim [m_D(\nu_k)]^2/R$ . This gives

$$\Gamma(\nu_i \rightarrow J \bar{\nu}_j) = \frac{h^2}{16\pi} \left[ \frac{m(\nu_i)}{R} \right]^4 m(\nu_j). \quad (10.5)$$

For  $h = 0.1$ ,  $R = 1$  TeV and  $m(\nu_j) = 65$  eV we get:

$$\tau(\nu_i) [m(\nu_i)]^4 = 5 \times 10^{34} \text{ eV}^4 \cdot \text{sec}. \quad (10.6)$$

The cosmological bound (2.19) gives for  $\nu_\mu$

$$m(\nu_\mu) \geq 13 \text{ MeV} \left[ \frac{0.1}{h} \right]^{1/4} \left[ \frac{R}{1 \text{ TeV}} \right]^{1/2} \left[ \frac{18 \text{ eV}}{m(\nu_e)} \right]^{1/8}. \quad (10.7)$$

Thus, with  $R \geq 1$  TeV we cannot have  $\nu_\mu$  heavier than 65 eV.

However, as the tree-level interactions of the majoron are with neutrinos only, there is no experimental or astrophysical lower bound on the scale of the lepton-number symmetry breaking (when taking into account higher-order processes, one may derive a very weak lower bound from stellar energy loss considerations [42]). We remarked in sect. 3 that  $R$  is most likely to be above 1 TeV, but that on the basis of presently available direct experimental information it can still be as low as 50 GeV. If one assumes that  $R$  is indeed 50 GeV (leading to  $\nu_\mu$  mass near its upper experimental bound), the majoron-emission decay of  $\nu_\mu$  may be consistent with the cosmological bound [47]. *If the global lepton-number symmetry is broken at a scale smaller than  $M_W$ , the decay (10.3) allows  $\nu_\mu$  with a mass between 70 keV and 250 keV.*

Note that the  $SU(2)_L \otimes U(1)_Y$ -singlet Higgs field  $S$  carrying  $L = -2$  with a vev below a TeV, cannot exist in LRS models or in any GUT (like  $SO(10)$ ,  $[SU(3)]^3$  or  $E_6$ ) which contains the LRS group. It can only exist as an ad-hoc field, invented for the sole purpose of allowing a majoron emission of a  $\nu_\mu$  with a mass close to its experimental limit. Note also that the above “window” for  $\nu_\mu$  is valid only if the cosmological limit (2.19) is saturated. If the universe is open with  $\Omega < 0.5$  or if its present age is at least  $1.2 \times 10^{10}$  y (rather than  $10^{10}$  y), the resulting strengthening of the cosmological bound prevents  $\nu_\mu$  decay even for  $R = 50$  GeV.

For  $\nu_\tau$  we get

$$m(\nu_\tau) \geq 11 \text{ MeV} \left[ \frac{0.1}{h} \right]^{1/4} \left[ \frac{R}{1 \text{ TeV}} \right]^{1/2} \left[ \frac{65 \text{ eV}}{m(\nu_j)} \right]^{1/8}. \quad (10.8)$$

Even if  $\nu_\mu$  has a mass at the upper experimental limit, namely 250 keV,  $\nu_\tau$  should be heavier than 4 MeV.

We conclude: *If global lepton-number symmetry breaking takes place at a scale larger than  $M_W$ ,  $\nu_\mu$  is lighter than 65 eV and  $\nu_\tau$  is either lighter than 65 eV or heavier than a few MeV.* If, however, the scale of global lepton-number symmetry is smaller than  $M_W$ , it is possible to have  $\nu_\mu$  between 70 keV and 250 keV.

## 10.2. GLOBAL HORIZONTAL SYMMETRY

The spontaneous breaking of a *global* horizontal symmetry implies massless Goldstone bosons, the “familons”. Neutrinos may decay [46] through

$$\nu_i \rightarrow f \nu_j, \quad (10.9)$$

where  $f$  is a familon. While estimating the decay rate, two important differences from the majoron emission decay should be taken into account:

(i) As familons carry no lepton number, they do not contribute to the Majorana mass of neutrinos. Consequently, there is no simultaneous diagonalization of the mass matrix and the familon couplings.

(ii) Charged leptons may decay into each other through familon emission. This gives a lower bound on the scale  $F$  of the familon interaction [43, 44]:

$$F \geq 10^{10} \text{ GeV}. \quad (10.10)$$

The coupling of the familon to light neutrinos of different generations is estimated [46] to be  $h \cdot [m(\nu_i)/F]$ . Here  $h$  is a Yukawa coupling. For  $F = 10^{10} \text{ GeV}$  and  $h = 0.1$  we get

$$\tau(\nu_i) [m(\nu_i)]^3 = 3.5 \times 10^{26} \text{ eV}^3 \cdot \text{sec}. \quad (10.11)$$

This and the cosmological constraints (2.19) give:

$$m(\nu_i) \geq 1.7 \text{ MeV} \left[ \frac{F}{10^{10} \text{ GeV}} \right]^2 \left[ \frac{0.1}{h} \right]^2. \quad (10.12)$$

For usual values of Yukawa couplings, the familon model does not allow  $\nu_\mu$  heavier than 65 eV. If we take  $h \geq 0.3$ , this mode may allow  $\nu_\mu$  with a mass near its upper experimental limit (for  $h = 1$ , we obtain  $m(\nu_\mu) \geq 17 \text{ keV}$ ).

We conclude: *The familon model does not allow  $\nu_\mu$  heavier than 65 eV, unless the Yukawa couplings of the familon are large ( $h \geq 0.3$ ).* The decay is fast enough (with  $F \sim 10^{10} \text{ GeV}$ ) for  $\nu_\tau$  heavier than a few MeV.

### 10.3. OTHER MODELS WITH GLOBAL SYMMETRIES

The *sequential lepton-number* model is an extension of the majoron model, designed to give faster neutrino decays [48]. The main ingredients are:

- (i) The leptons in each generation have a different lepton number  $l_i$ .
- (ii) There are several Higgs fields (we study only the multiple singlets model), each with a different lepton number  $L_j$ .

We find that only very restricted choices of the lepton numbers  $L_n$  allow this decay mode (various combinations lead to either a degeneracy between neutrinos, or purely diagonal majoron couplings). Just as in the original majoron model, if the light neutrino is exactly massless, it decouples from the majoron. We obtain the following relation (for  $m(\nu_j) = 65 \text{ eV}$ ,  $h = 0.1$ )

$$\tau(\nu_i) [m(\nu_i)]^2 = 5 \times 10^{10} \left[ \frac{R}{1 \text{ TeV}} \right]^2 \text{ eV}^2 \cdot \text{sec}. \quad (10.13)$$

Consequently, the cosmological bound sets *an upper bound on the scale of lepton-number breaking*,  $R \leq 6 \times 10^7 \text{ GeV}$ . *As the majoron does not couple to charged leptons, there is no other detectable implication of the model.*

Several other models with global symmetry breaking were suggested: a  $[U(1)]^3$  symmetry group carried by leptons and additional Higgs fields [49], a  $U(1)$  group

carried by only Higgs doublets added to the LRS model [50], etc. They all require a similar degree of complexity.

Our overall conclusion in this section is that neutrino decays into majorons or familons may be consistent with the cosmological bounds only when we assume very unusual values of the parameters (e.g.  $R = 50$  GeV or  $h \sim 1$ ) or when extremely complicated ad-hoc assumptions are invoked.

## 11. Additional constraints from the see-saw mechanism

In all models discussed so far (except for the unlikely ad-hoc schemes mentioned in sect. 10), we could not have  $\nu_\mu$  heavier than 65 eV and  $\nu_\tau$  with a mass above 65 eV and below 900 keV. These bounds are independent of the specific form of the neutrino mass matrix. In this section we consider possible additional constraints which may be imposed on neutrino masses by the see-saw mechanism.

### 11.1. MASS RATIOS AMONG NEUTRINOS AND THE “REASONABLE SEE-SAW”

For the sake of definiteness, we study the see-saw mechanism in the minimal LRS model described in sect. 6. However, our conclusions are quite general. The neutrino mass matrix is given in eq. (6.1):

$$M = \begin{pmatrix} -hv_L & \frac{1}{2}(h_1 k_1 + h_2 k_2) \\ \frac{1}{2}(h_1^T k_1 + h_2^T k_2) & hv_R \end{pmatrix}. \quad (11.1)$$

The  $h$ 's are Yukawa coupling *matrices*. As mentioned earlier

- (i)  $v_L \sim k^2/v_R$  is very small and we neglect it.
- (ii) The Dirac mass matrix of the charged leptons is  $\frac{1}{2}(h_2 k_1 + h_1 k_2)$ , and is expected to be of the same order of magnitude as  $m_D(\nu)$ . For simplicity we replace  $m_D(\nu) \rightarrow m_D(\ell)$ . (We note that this assumption is not better than a similar assumption on the up and the down quark masses. We remember that the masses of the two quarks in the same doublet may differ even by an order of magnitude.)

We do not know the form of the  $h$ -matrices. Two “reasonable” possibilities are:

- (i) The new physics that leads to the Majorana mass matrix  $M_R$  is “blind” to whatever mechanism which is responsible for the mass hierarchy among generations. In the basis where  $M_R$  is diagonal this means

$$h_e \sim h_\mu \sim h_\tau. \quad (11.2)$$

If  $m_D$  and  $M_R$  can be diagonalized simultaneously we get  $m(\nu_i) \sim [m(\ell_i)]^2/R$ , and in particular:

$$\frac{m(\nu_\tau)}{m(\nu_\mu)} \sim \left[ \frac{m(\tau)}{m(\mu)} \right]^2. \quad (11.3)$$



(ii) The mechanism that gives the mass hierarchy among generations in  $m_D$  acts in a similar way in  $M_R$ . In the basis where  $M_R$  is diagonal this gives

$$h_e : h_\mu : h_\tau \propto m(e) : m(\mu) : m(\tau). \quad (11.4)$$

If  $m_D$  were diagonal at the same time, mass ratios between neutrinos would be similar to those between charged leptons, and in particular:

$$\frac{m(\nu_\tau)}{m(\nu_\mu)} \sim \frac{m(\tau)}{m(\mu)}. \quad (11.5)$$

In the general case,  $m_D$  and  $M_R$  cannot be simultaneously diagonalized. However, it turns out that in most cases, a “reasonable see-saw” matrix, namely one that follows either of the assumptions (i) and (ii) gives:

$$\frac{m(\nu_i)}{m(\nu_j)} \sim \left[ \frac{m(\ell_i)}{m(\ell_j)} \right]^p \quad \text{with } 1 \leq p \leq 2. \quad (11.6)$$

In order to have  $p > 2$  we need, in general, a matrix  $M_R$  with an inverted hierarchy, e.g.  $h_\tau/h_\mu \sim m(\mu)/m(\tau)$ . We do not know any sensible model with such a prediction, but we cannot completely exclude it and we discuss this possibility in subsect. 11.2.

The bound  $p \geq 1$  is somewhat less certain for the following reasons:

(i) Neglecting the contribution of  $v_L$  may be unjustified. For example, the mass differences among different generations may result from different vev's of several  $\phi$ 's. If there is just one Higgs triplet, we generally have  $v_L \sim k_{\max}^2/v_R$ . In such a case, all the light neutrinos masses are expected to be of the same order of magnitude,  $m(\nu) \sim [m(\tau)]^2/R$ . Consequently, we may obtain  $p = 0$ .

(ii) If  $M_R$  vanishes at tree level but gets a large contribution at the one- or two-loop level, then it may depend on Yukawa couplings of  $h_{1,2}$ . This may lead to  $h_i \propto [m(\ell_i)]^q$  with  $q \geq 1$ . For example [51], in the  $[SU(3)]^3 \otimes Z_3$  GUT,  $q = 3$ , leading to an inverted hierarchy among the light neutrinos and to  $p = -1$ .

We conclude that  $1 \leq p \leq 2$  is “reasonable” and likely, but cannot be proven. The  $p \leq 2$  assumption is somewhat more solid than the  $p \geq 1$  assumption. In the following sections we will refer to the inequality (11.6) as the “reasonable see-saw” assumption.

## 11.2. HOW DIFFICULT IS IT TO EVADE THE “REASONABLE SEE-SAW” ASSUMPTION?

Before proceeding to derive significant new results with the aid of the “reasonable see-saw” assumption, we wish to consider examples of schemes which manage to evade this assumption. By studying the complexities involved in such schemes, we can get a good feeling for the validity of the assumption.

The “reasonable see-saw” assumption (eq. (11.6)) consists of two inequalities:  $p \geq 1$  and  $p \leq 2$ . In the previous section we discussed ways of avoiding  $p \geq 1$ . Since our strongest results will depend mainly on the  $p \leq 2$  assumption, we will now consider several scenarios which actually lead to  $p > 2$ . We will see that, while such scenarios cannot be ruled out mathematically, we cannot really find any good physical motivation for them. We will therefore maintain our conclusion that  $p \leq 2$  is very likely but cannot be proven.

Our first scenario refers to the case in which  $m_D$  and  $M_R$  can be simultaneously diagonalized. In that case, we have:

$$\frac{h_i}{h_j} = \left[ \frac{m(\ell_i)}{m(\ell_j)} \right]^{2-p}, \quad (11.7)$$

where  $h_i, h_j$  are the Yukawa couplings in the Majorana masses of the  $i, j$  generations. For  $p > 2$  this requires an “inverted hierarchy” in the Majorana masses. For instance, in order to obtain:

$$\frac{m(\nu_\tau)}{m(\nu_\mu)} \sim \left[ \frac{m(\tau)}{m(\mu)} \right]^3, \quad (11.8)$$

we need:

$$\frac{M_R(\nu_\tau)}{M_R(\nu_\mu)} \sim \frac{m(\mu)}{m(\tau)}. \quad (11.9)$$

We are not aware of any simple “see-saw” which would lead to such an inversion. The only scenario we can offer for it involves an extended type of “see-saw” which actually appears in some GUTs [52] and string inspired models [53]. In such theories,  $\nu_R$  does not acquire a Majorana mass, but there are additional fermions which are singlets of the gauge group and may acquire Majorana masses. This gives an “extended” see-saw matrix of the form:

$$M = \begin{pmatrix} 0 & m_D & 0 \\ m_D^T & 0 & M_D \\ 0 & M_D^T & M_X \end{pmatrix}, \quad (11.10)$$

where  $m_D$ ,  $M_D$  and  $M_X$  may correspond to three different mass scales. For the purpose of evaluating the light neutrino masses, we can replace this “extended see-saw” by a simple effective see-saw matrix  $\begin{pmatrix} 0 & m_D \\ m_D^T & M_R \end{pmatrix}$  with the substitution:

$$M_R = -M_D M_X^{-1} M_D^T. \quad (11.11)$$

The light neutrinos mass matrix is then

$$m_D (M_D^T)^{-1} M_X (M_D)^{-1} m_D^T. \quad (11.12)$$

If, for some reason,  $M_X$  has a hierarchy among generations (as in  $m_D$ ), but  $M_D$  is “generation-blind”, then the *effective*  $M_R$  has an *inverted* hierarchy and we end up with  $p = 3$ . On the other hand, if both  $M_D$  and  $M_X$  have no hierarchy, or if both have a similar hierarchy, the “reasonable see-saw” remains valid. Even with this complicated scenario which may allow  $p = 3$ , we find it difficult to imagine reasons for obtaining  $p > 3$  values. This will become important in subsect. 11.3.

Our second scenario relates to the more likely case in which the submatrices  $m_D$  and  $M_R$  cannot be simultaneously diagonalized. We have to assume some explicit form for the non-diagonal mass matrices. An attractive (though not unique) possibility is the Fritzsch form [54] (for simplicity we assume a symmetric Yukawa matrix):

$$m_D = \begin{pmatrix} 0 & h_{e\mu} k_3 & 0 \\ h_{e\mu} k_3 & 0 & h_{\mu\tau} k_1 \\ 0 & h_{\mu\tau} k_1 & h_{\tau\tau} k_0 \end{pmatrix}, \quad (11.13)$$

where  $k_i$  are vev's of different Higgs fields  $\phi_i$  and  $h_{ij}$  are Yukawa couplings.

This form may arise in a variety of models and is presently consistent with all data on quark masses and mixing angles. Typically, we have a hierarchy of the form

$$h_{\tau\tau} k_0 > h_{\mu\tau} k_1 > h_{e\mu} k_3. \quad (11.14)$$

The simplest way of obtaining the Fritzsch form is to assume that fermions as well as Higgs fields carry a spontaneously broken “generation-number” (which we label  $G$ ). In such a case, Yukawa couplings that do not conserve  $G$  vanish. For example, if  $G_e$ ,  $G_\mu$  and  $G_\tau$  are 2, 1 and 0 respectively, and there are three Higgs doublets with  $G = 0, 1$  and 3 that obtain vev's  $k_0, k_1, k_3$  as in eq. (11.13), we obtain a Fritzsch form. The hierarchy is completely determined by  $k_0 > k_1 > k_3$  and all Yukawa couplings may be of the same order of magnitude. The same type of matrix arises when the fermion  $G$ -values are  $-1, +1$  and 0 with only two Higgs doublets carrying  $G = 0$  and 1, but then there should be some hierarchy among the Yukawa couplings as well.

If we assume that  $m_D$  and  $M_R$  have Fritzsch forms, in which all matrix elements are either comparable to each other or obeying a hierarchy similar to eq. (11.14), we usually obtain light neutrino masses which are consistent with the “reasonable see-saw” assumption.

We have searched through all possible matrices of this type and found only several artificial examples in which, by making ad-hoc assumptions we could extract

neutrino mass ratios which violate the “reasonable see-saw” hypothesis and yield  $p \sim 3$  values. These examples are described in appendix B.

After considering a large variety of possibilities we therefore conclude that avoiding the “reasonable see-saw” is artificial, unlikely but not impossible. We now proceed to study the consequences of the “reasonable see-saw”.

### 11.3. CONSEQUENCES OF “REASONABLE SEE-SAW” MATRICES

As we have shown, the cosmological bound on the energy density of the universe can be fulfilled only if

$$m(\nu_\mu) \leq 65 \text{ eV}, \quad m(\nu_\tau) \leq 65 \text{ eV} \quad \text{or} \quad m(\nu_\tau) \geq 0.9 \text{ MeV}. \quad (11.15)$$

On the other hand, the “reasonable see-saw” assumption puts an upper limit on the mass ratio (the  $p \sim 2$  limit of eq. (11.6)):

$$\frac{m(\nu_\tau)}{m(\nu_\mu)} \leq \left[ \frac{m(\tau)}{m(\mu)} \right]^2 \sim 300. \quad (11.16)$$

However, if  $\nu_\tau$  is heavier than 0.9 MeV, the same mass ratio must obey  $m(\nu_\tau)/m(\nu_\mu) \geq 14000$ , demanding  $p \geq 3.4$ , in clear conflict with the “reasonable see-saw”. We have seen in the previous section that  $p \sim 3$  values are quite unlikely and  $p > 3$  is even less plausible. *This leads to the conclusion*

$$m(\nu_\mu) \leq 65 \text{ eV}, \quad m(\nu_\tau) \leq 65 \text{ eV}. \quad (11.17)$$

*If neutrinos are light as a consequence of a “reasonable see-saw” mechanism, then it is impossible to accommodate the cosmological constraints on their masses, unless they are all lighter than 65 eV.*

The strong limits obtained in eq. (11.17) have further implications. The lower bound on the mass ratio among neutrinos (the  $p \geq 1$  limit of eq. (11.6)) can be combined with  $m(\nu_\tau) \leq 65 \text{ eV}$  to give:

$$m(\nu_\mu) \leq 4 \text{ eV}, \quad m(\nu_e) \leq 0.02 \text{ eV}. \quad (11.18)$$

Thus, the “reasonable see-saw” hypothesis, together with our previous conclusions, leads us to an extremely strong new upper bound on the masses of  $\nu_\tau$ ,  $\nu_\mu$  and  $\nu_e$ .

As the mass of  $\nu_\tau$  is assumed to be approximately given by  $m(\nu_\tau) = [m(\tau)]^2/R$ , the above upper bound on  $m(\nu_\tau)$  gives a *lower* bound on the scale  $R$ :

$$R \geq \frac{[m(\tau)]^2}{65 \text{ eV}} \sim 50 \text{ PeV}. \quad (11.19)$$

This is a very significant bound if  $R$  is the scale of LRS breaking or of a horizontal gauge-symmetry breaking. This bound is not significant for GUTs, in which the breaking-scale is known to be much higher.

We can see only two possible ways of evading the conclusions (11.17)–(11.19):

(i) Avoid the “reasonable see-saw” assumption, so that eq. (11.6) is not valid. We discussed this possibility in subsect. 11.2 and in appendix B.

(ii) Find additional decay channels for  $\nu_\mu$  and/or  $\nu_\tau$ . Such decay modes should either have no relation to analogous decays of charged leptons, or be heavily suppressed in the charged sector. In this way, eq. (11.15) may be circumvented. We have discussed such cases in sect. 10.

## 12. A fourth leptonic generation

### 12.1. $\nu_\mu$ AND $\nu_\tau$ DECAYS

One of the simplest extensions of the standard model is the addition of a fourth fermionic generation. We label the fourth charged lepton by  $\sigma$ , and the corresponding neutrino by  $\nu_\sigma$ . As yet, there is no experimental evidence for their existence. There is a direct experimental lower limit [55]  $m(\sigma) \geq 41$  GeV. The  $\rho$ -parameter measurement puts an upper limit [56] on the mass splitting between  $\sigma$  and  $\nu_\sigma$ : if  $\nu_\sigma$  is very light,  $\sigma$  cannot be heavier than 300 GeV. If the fourth generation neutrino  $\nu_\sigma$  is sufficiently heavy (e.g.  $m(\nu_\sigma) \geq 4.2$  GeV) it can be stable or unstable (see eq. (2.14)). Still, the existence of a fourth generation could affect the decays of lighter neutrinos. We first study these effects, and calculate whether our former conclusions remain valid.

The existence of a fourth generation enhances the radiative decay  $\nu_i \rightarrow \nu_j \gamma$  for  $i = \mu, \tau$  [31]. The reason is that as long as all charged leptons masses are much smaller than  $M_W$ , this decay channel is suppressed by a GIM mechanism. The existence of a charged lepton with a mass comparable to  $M_W$  eliminates the GIM suppression. To see this effect, we note that in the expression for the decay width (eq. (4.13)),

$$\Gamma(\nu_i \rightarrow \nu_j \gamma) = \frac{\alpha G_F^2 [m(\nu_i)]^5}{64\pi^4} \left[ \sum_a U_{ja} U_{ia} F(r_a) \right]^2, \quad (12.1)$$

the term in square brackets is approximately given by  $U_{i\sigma} U_{j\sigma} [F(r_\sigma) - F(0)]$ . The function  $[F(r_\sigma) - F(0)]$  varies slowly from  $\frac{1}{4}$  to  $\frac{3}{4}$  as  $r_\sigma$  varies from 1 to infinity. In the limit that  $r_\sigma$  approaches infinity, the lifetime for the radiative decay is [31]:

$$\tau(\nu_i) [m(\nu_i)]^5 = (U_{i\sigma} U_{j\sigma})^{-2} 10^{36} \text{ eV}^5 \cdot \text{sec}. \quad (12.2)$$

The requirement for radiative lifetime shorter than  $10^4$  sec, cannot be satisfied for  $\nu_\mu$ . In the  $\nu_\tau$  case, we are again led to the range of masses above 10 MeV.

In all other processes, the existence of a heavier leptonic doublet does not affect the decay rate. Thus, our former conclusions remain valid:  $\nu_\mu$  is lighter than 65 eV;  $\nu_\tau$  can be heavier than 0.9 MeV (if it is unstable), but if we assume a “reasonable see-saw”,  $m(\nu_\tau) \leq 65$  eV as well.

## 12.2. $\nu_\sigma$ DECAYS WITHIN THE STANDARD MODEL

As mentioned before,  $\nu_\sigma$  could be stable or unstable, with a mass larger than 4.2 GeV. However, if we make the “reasonable see-saw” assumption we obtain

$$m(\nu_\sigma) \leq \left[ \frac{m(\sigma)}{m(\tau)} \right]^2 m(\nu_\tau). \quad (12.3)$$

This is exactly the assumption which led us to  $m(\nu_\tau) \leq 65$  eV. The charged lepton mass is bounded by the  $\rho$ -measurement,  $m(\sigma) \leq 300$  GeV. Putting these limits into eq. (12.3) gives

$$m(\nu_\sigma) \leq 2 \text{ MeV}. \quad (12.4)$$

Can we have a  $\nu_\sigma$  with a mass larger than 65 eV and lighter than 2 MeV? To answer this question, we repeat our analysis for the various possible  $\nu_\sigma$  decay modes.

The rate of the Z-mediated decay into three lighter neutrinos depends on the coupling of Z to neutrinos of different generations, which is  $O(D^2/R^2)$ . As  $m(\sigma) \leq 300$  GeV, and  $R \geq 50$  PeV (eq. (11.19)),  $D^2/R^2 \leq 10^{-10}$ , which makes this channel irrelevant to our discussion.

The W-mediated decay into  $e^+e^-\nu_e$  depends on the  $\nu_e$ - $\nu_\sigma$  mixing. The same upper bounds as in the  $\nu_\tau$  case (eq. (4.12)) apply here and, consequently, the decay is too slow for  $m(\nu_\sigma) < 10$  MeV.

The radiative decay lifetime is given (for  $\nu_\sigma$  lighter than 2 MeV) by eq. (12.2) with  $i = \sigma$ . The double photon decay is given by eq. (4.17). Both are much too slow to allow a  $\nu_\sigma$  in the mass range in question.

We conclude that in the extended standard model (with right-handed neutrinos and fourth generation fermions) with a “reasonable see-saw” mass matrix,  $m(\nu_\sigma) \leq 65$  eV.

## 12.3. $\nu_\sigma$ DECAY IN LRS AND HORIZONTAL MODELS

The fourth neutrino  $\nu_\sigma$  may still be heavier than 65 eV and lighter than 2 MeV, if it decayed into three neutrinos through one of the channels described in sects. 6 and 7. We cannot rule out such a decay, because we have no upper limit on the decay of the hypothetical charged  $\sigma$  into three charged leptons. However, we can reverse the argument, assume  $m(\nu_\sigma) > 65$  eV, and study the implications on charged leptons decays. In the models of sects. 6 and 7, there is a relation between  $\nu_\sigma$  and  $\sigma$  decays:

$$\frac{\Gamma(\sigma \rightarrow 3\ell)}{\Gamma(\nu_\sigma \rightarrow 3\nu)} = \left[ \frac{m(\sigma)}{m(\nu_\sigma)} \right]^5. \quad (12.5)$$

We are interested in the branching ratio

$$B \equiv \frac{\Gamma(\sigma \rightarrow 3\ell)}{\Gamma(\sigma \rightarrow e\nu\nu)}, \quad (12.6)$$

where  $\sigma \rightarrow e\nu\bar{\nu}$  is the “normal”, W-boson mediated decay which occurs in the standard model. We have:

$$\frac{\Gamma(\sigma \rightarrow e\nu\nu)}{\Gamma(\tau \rightarrow e\nu\nu)} = \left[ \frac{m(\sigma)}{m(\tau)} \right]^5. \quad (12.7)$$

The three equations (12.5), (12.6) and (12.7) together give:

$$B = \frac{\Gamma(\nu_\sigma \rightarrow 3\nu)}{\Gamma(\tau \rightarrow e\nu\nu)} \left[ \frac{m(\tau)}{m(\nu_\sigma)} \right]^5. \quad (12.8)$$

If  $m(\nu_\sigma) > 65$  eV, the  $3\nu$  final states provide the main decay modes and thus  $\Gamma(\nu_\sigma \rightarrow 3\nu) = [\tau(\nu_\sigma)]^{-1}$ . The  $\nu_\sigma$  lifetime must fulfill the cosmological bound (2.19):

$$[m(\nu_\sigma)]^2 \tau(\nu_\sigma) \leq 2 \times 10^{20} \text{ eV}^2 \cdot \text{sec} \quad (12.9)$$

and we assume the “reasonable see-saw” relation (12.3):

$$m(\nu_\sigma) \leq \left[ \frac{m(\sigma)}{m(\tau)} \right]^2 m(\nu_\tau), \quad (12.10)$$

with  $m(\nu_\tau) \leq 65$  eV. Using eqs. (12.8), (12.9) and (12.10) we then obtain a *lower* bound:

$$\frac{\Gamma(\sigma \rightarrow 3\ell)}{\Gamma(\sigma \rightarrow e\nu\nu)} \geq 3.6 \left[ \frac{41 \text{ GeV}}{m(\sigma)} \right]^6. \quad (12.11)$$

This leads to an overall branching ratio  $\Gamma(\sigma \rightarrow 3\ell)/\Gamma(\sigma \rightarrow \text{all})$  between 28% (for  $m(\sigma) = 41$  GeV,  $m(\nu_\sigma) > 65$  eV) and 0.5% (for  $m(\sigma) \sim M_W$ ,  $m(\nu_\sigma) > 65$  eV). A branching ratio of the order of 28% at  $m(\sigma) \sim 41$  GeV would have probably been observed by the UA1 detector and is presumably already ruled out.

We conclude: the “reasonable see-saw” assumption leads to the conclusion that  $m(\nu_\sigma) \leq 2$  MeV. If  $m(\nu_\sigma) > 65$  eV, it should decay into  $3\nu$  through  $\Delta_L$  or  $H^a$  exchange. *The decay of  $\sigma$  into three charged leptons is then an important decay mode.*

If, however, a fourth generation lepton is observed and its branching ratio for the  $3\ell$  final state is found to be smaller than the bound (12.11), we will be led to conclude that  $\nu_\sigma$  is also lighter than 65 eV. In that case, the “reasonable see-saw” assumption will allow us to further decrease the upper bounds for  $m(\nu_\tau)$ ,  $m(\nu_\mu)$  and  $m(\nu_e)$ . All three bounds (eqs. (11.17), (11.18)) become smaller by a common factor  $m(\tau)/m(\sigma)$ .

## 12.4. A FOURTH GENERATION IN THE MAJORON SCHEME

In all “beyond standard” models with a scale above 1 TeV, the “reasonable see-saw” assumption leads to the conclusion that  $\nu_e$ ,  $\nu_\mu$  and  $\nu_\tau$  are lighter than 65 eV, while  $\nu_\sigma$  is lighter than 2 MeV. In sect. 10 we mentioned that if one assumes that global lepton-number symmetry is broken at a scale as low as 50 GeV, the neutrinos can have masses near the upper experimental bounds [47]. If experiments find  $m(\sigma) \sim 50$  GeV, the mass of  $\nu_\sigma$  is predicted to be  $O(50 \text{ GeV})$  as well. Such a neutrino can be stable or unstable without violating the cosmological bounds.

We conclude: In the majoron scheme, with the masses of all three known neutrinos near their upper experimental bounds [47], *it is possible that all leptons of the fourth generation ( $\sigma, \nu_{\sigma L}, \nu_{\sigma R}$ ) have their masses around 50 GeV.*

## 13. Conclusions

In our analysis we have used *five* different ingredients:

- (i) Direct experimental bounds on neutrino masses.
- (ii) Cosmological bounds on the masses of stable neutrinos and on the relation between the masses and the lifetimes of unstable neutrinos.
- (iii) Theoretical calculations of neutrino decay rates and their relations to neutrino masses in models in which all relevant parameters are known (particularly the standard model, but also some “beyond standard” models).
- (iv) Experimental bounds on specific decays of *charged* leptons and theoretical relations between such decays and neutrino decays. These relations are helpful in models (i.e. LRS-theory, horizontal symmetry, substructure) in which the relevant parameters are not known.
- (v) The “reasonable see-saw” assumption.

The first four ingredients are based on experimental data, on the standard cosmological model, on the standard electroweak theory and on very conservative and general assumptions concerning “beyond standard” models. We consider these ingredients to be very reliable. The fifth ingredient (“reasonable see-saw”) is slightly less solid, but is still likely to be valid.

By combining the first *four* ingredients we conclude:

$$\begin{aligned} m(\nu_e) &\leq 18 \text{ eV}, \\ m(\nu_\mu) &\leq 65 \text{ eV}, \\ m(\nu_\tau) &\leq 65 \text{ eV} \quad \text{or} \quad 0.9 \text{ MeV} \leq m(\nu_\tau) \leq 70 \text{ MeV}. \end{aligned} \tag{13.1}$$

By combining *all five* ingredients we conclude:

$$\begin{aligned} m(\nu_e) &\leq 0.02 \text{ eV}, \\ m(\nu_\mu) &\leq 4 \text{ eV}, \\ m(\nu_\tau) &\leq 65 \text{ eV}. \end{aligned} \tag{13.2}$$



This last conclusion has additional important implications. A see-saw mechanism together with an *upper bound* on a left-handed neutrino mass, imply a *lower limit* on the Majorana mass of the corresponding right-handed neutrino:

$$M(\nu_R) \sim \frac{[m(\tau)]^2}{m(\nu_\tau)} \geq 50 \text{ PeV}. \quad (13.3)$$

(1 PeV =  $10^3$  TeV). For a “see-saw” driven by the GUT scale or the Planck scale, this bound is useless. However, for LRS theories it implies (assuming  $h_{\Delta\nu\nu} \leq g_{\text{weak}}$ ):

$$M(W_R) \geq 50 \text{ PeV} \quad (13.4)$$

and for a “see-saw” driven by a horizontal symmetry we obtain:

$$M(H^a) \geq 50 \text{ PeV}. \quad (13.5)$$

Both of these limits are very significant. In the case of LRS theories they imply that no right-handed W or Z will be produced in experiments within the next several decades and that most effects (including *CP* violation) which are due to right-handed currents are negligible. Previous bounds on the scale of right-handed currents [57–59] were in the range of a few TeV, well below our new bound. In the case of horizontal symmetry, the new bound is stronger than previous bounds [60, 38] obtained from rare processes such as  $\mu N \rightarrow e N$ ,  $\mu \rightarrow 3e$ ,  $\mu \rightarrow e\gamma$ ,  $K^0 \rightarrow e\mu$ ,  $K^+ \rightarrow \pi^+ \mu e$  and  $\Delta M(K_S^0 - K_L^0)$ .

All our bounds seem to be valid within the framework of all currently popular “beyond standard” models (with the possible exception of the ad-hoc schemes mentioned in sect. 10). We believe that these bounds are theoretically significant. The upper bounds on  $m(\nu_\tau)$ ,  $m(\nu_\mu)$ ,  $m(\nu_e)$  are, respectively, six, five and three orders of magnitudes below the corresponding experimental bounds. The lower bound on  $M(W_R)$  is four orders of magnitude above the previous bounds. From the pure experimental point of view, these bounds imply that direct experiments in the foreseeable future have no chance of observing neutrino masses or right-handed W-bosons.

Can we expect additional information from experiments in the next few years? Improvements in the direct bound on  $m(\nu_\mu)$  may help eliminate the small “window” allowed by the model of ref. [47]. If a fourth generation lepton is discovered and if it does not decay to three charged leptons, we may obtain bounds which are even stronger than eq. (13.2). Improvements of the bounds on  $\tau \rightarrow 3\ell$  may strengthen our confidence in the consequences of the “reasonable see-saw”.

The resulting range of allowed neutrino masses is perfectly consistent with models which produce a see-saw based on the Planck scale, the GUT scale or the so-called intermediate scale ( $\sim 10^{11}$  GeV). It is also consistent with the neutrino mass range

required for explaining the solar neutrino puzzle in terms of resonant neutrino oscillations in matter [61].

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## Appendix A

### THE HIGGS SECTOR IN THE MINIMAL LRS MODEL

We study the masses of the Higgs particles in the minimal LRS model [25]. The Higgs sector consists of:

$$\begin{aligned}\Phi &= \left(\frac{1}{2}, \frac{1}{2}\right)_0 = \begin{pmatrix} \phi_1^0 & \phi_1^+ \\ \phi_2^- & \phi_2^0 \end{pmatrix}, \\ \Delta_L &= (1, 0)_2 = \begin{pmatrix} \sqrt{\frac{1}{2}} \Delta_L^+ & \Delta_L^{++} \\ \Delta_L^0 & -\sqrt{\frac{1}{2}} \Delta_L^+ \end{pmatrix}, \\ \Delta_R &= (0, 1)_2 = \begin{pmatrix} \sqrt{\frac{1}{2}} \Delta_R^+ & \Delta_R^{++} \\ \Delta_R^0 & -\sqrt{\frac{1}{2}} \Delta_R^+ \end{pmatrix}.\end{aligned}\tag{A.1}$$

The vev's of the Higgs fields are of the general form:

$$\begin{aligned}\langle \Phi \rangle &= \begin{pmatrix} k_1 & 0 \\ 0 & k_2 \end{pmatrix}, \\ \langle \Delta_L \rangle &= \begin{pmatrix} 0 & 0 \\ v_L & 0 \end{pmatrix}, \\ \langle \Delta_R \rangle &= \begin{pmatrix} 0 & 0 \\ v_R & 0 \end{pmatrix}.\end{aligned}\tag{A.2}$$

The four vev's are complex, in general. However, by an appropriate  $SU(2)_L \otimes SU(2)_R$  transformation, we can make  $k_1 \cdot k_2^*$  and  $v_L \cdot v_R$  real. We define  $\xi \equiv \arg(k_i)$  and  $\eta \equiv \arg(v_L)$ . From now on, we denote by  $k_1, k_2, v_L, v_R$  the absolute values of the vev's. At the minimum of the potential:

$$v_L = - \frac{(\gamma_{11} + \gamma_{22})k_1 k_2 \cos(2\eta) + \gamma_{12} \{ k_1^2 \cos[2(\eta + \xi)] + k_2^2 \cos[2(\eta - \xi)] \}}{[\rho_3 - 2(\rho_1 + \rho_2)] v_R},\tag{A.3}$$

where the  $\gamma$ 's and the  $\rho$ 's are coefficients in the Higgs potential defined in ref. [25]. Thus, assuming  $v_R^2 \gg k^2 \equiv k_1^2 + k_2^2$ , we naturally get  $v_L^2 \ll k^2$ . The mass matrix for the charged gauge bosons is:

$$[M(W^\pm)]^2 = \frac{1}{2}g^2 \begin{pmatrix} 2v_L^2 + k^2 & -2k_1k_2 \\ -2k_1k_2 & 2v_R^2 + k^2 \end{pmatrix}. \quad (\text{A.4})$$

Consequently,  $M(W_R) \gg M(W_L)$  and the mixing between them,  $\xi = k_1k_2/v_R^2$ , is very small even if one does not assume  $k_2/k_1 \ll 1$  [59].

We first consider the masses of the Higgs particles [62] that arise only from the  $SU(2)_R$ -breaking (i.e.  $k^2 = v_L^2 = 0$ ). As the  $\Delta_L$ -triplet is an  $SU(2)_R$  singlet, its three members are degenerate in mass:

$$[M(\Delta_L)]^2 = \rho v_R^2, \quad (\text{A.5})$$

where  $\rho \equiv \rho_3 - 2(\rho_1 + \rho_2)$ . With no fine-tuning, we expect  $\rho = O(1)$ , and thus the  $\Delta_L$ -triplet has its mass at the scale of  $SU(2)_R$ -breaking (the consequences of assuming  $\rho < 10^{-8}$  are discussed in ref. [63]).

The masses of the right-handed triplet members are:

$$[M(\Delta_R^{++})]^2 = -2\rho_2 v_R^2; \quad [M(\text{Re}[\Delta_R^0])]^2 = 4(\rho_1 + \rho_2) v_R^2. \quad (\text{A.6})$$

$\Delta_R^+$  and  $\text{Im}[\Delta_R^0]$  are the massless Goldstone bosons. The mass matrix for the neutral  $\phi$  fields is:

$$[M(\phi^0)]^2 = v_R^2 \begin{pmatrix} 0 & 0 \\ 0 & (\beta_{22} - \beta_{11}) \frac{k_1^2 + k_2^2}{k_1^2 - k_2^2} \end{pmatrix}, \quad (\text{A.7})$$

where the  $\beta$ 's are defined in ref. [25]. The mass matrix for the charged  $\phi$  fields is similar to the matrix (A.7). We now "switch-on" the vev's  $k_1$  and  $k_2$ , and study the new mass terms that arise to  $O(kv_R)$ :

$\Delta_L^{++}$ : The mass of the doubly-charged Higgs gets no corrections to this order. Thus:

$$[M(\Delta_L^{++})]^2 = \rho v_R^2 + O(k^2). \quad (\text{A.8})$$

$\Delta_L^+$ : The singly-charged Higgs field mixes with the  $\phi^+$  fields. The mixing term is a function of all the  $\gamma_{ij}$  coefficients,  $f(\gamma)kv_R$ . The mass-squared difference between

$\Delta_L^+$  and  $\phi^+$  is a function of all the  $\rho_i$  and  $\beta_{ii}$  coefficients,  $g(\rho, \beta)v_R^2$ . Thus,

$$[M(\Delta_L^+)]^2 = \begin{cases} \rho v_R^2 + O(kv_R) & \text{if } \frac{g}{f} \leq \frac{k}{v_R} \\ \rho v_R^2 + O(k^2) & \text{otherwise.} \end{cases} \quad (\text{A.9})$$

$\Delta_L^0$ : The neutral Higgs field mixes with the  $\phi^0$  fields. The mixing term is related to the mixing term for the  $\Delta_L^+$  through a Clebsch-Gordan coefficient. The mass differences are exactly the same in both cases. Thus, our conclusions are similar to (A.9).

We conclude: *Assuming that all the coefficients in the Higgs potential are  $O(1)$ , and that there are no accidental cancellations among them, the mass splittings within the  $\Delta_L$  triplet are:*

$$\frac{|[M(\Delta_L^{++})]^2 - [M(\Delta_L^0)]^2|}{[M(\Delta_L^{++})]^2} \sim O\left[\frac{k^2}{v_R^2}\right] \sim O\left[\frac{M(W_L)}{M(W_R)}\right]^2 < 2.5 \times 10^{-3}. \quad (\text{A.10})$$

If we fine-tune the Higgs potential parameters to  $O(k/v_R)$ , we still get:

$$\frac{|[M(\Delta_L^{++})]^2 - [M(\Delta_L^0)]^2|}{[M(\Delta_L^{++})]^2} \sim O\left[\frac{k}{v_R}\right] \sim O\left[\frac{M(W_L)}{M(W_R)}\right] < 5 \times 10^{-2}. \quad (\text{A.11})$$

## Appendix B

### GENERATION NUMBERS AND EXCEPTIONAL MASS RATIOS

We survey matrices  $M_R$  with *no hierarchy*, but with *special forms* dictated by some horizontal (discrete, global or local) symmetry, which may give  $m(\nu_\tau)/m(\nu_\mu) > [m(\tau)/m(\mu)]^2$ .

As discussed in subsect. 11.2, we assume that fermions as well as Higgs fields carry a spontaneously broken “generation number” which we label  $G$ . For the sake of definiteness, we take  $G_e$ ,  $G_\mu$  and  $G_\tau$  to be 2, 1 and 0 respectively, and assume that there are three Higgs doublets with  $G = 0, 1$  and 3, that obtain vev’s  $k_0$ ,  $k_1$  and  $k_3$ , respectively. Similar results are obtained from other sets of  $G$ -values. The resulting mass matrix is of the Fritzsch form [54]

$$m_D = \begin{pmatrix} 0 & h_{e\mu}k_3 & 0 \\ h_{e\mu}k_3 & 0 & h_{\mu\tau}k_1 \\ 0 & h_{\mu\tau}k_1 & h_{\tau\tau}k_0 \end{pmatrix}. \quad (\text{B.1})$$

We first study matrices  $M_R$  with three eigenvalues different from zero. We find that a *necessary* condition for  $m(\nu_\tau)/m(\nu_\mu) > [m(\tau)/m(\mu)]^2$  is that the matrix elements of  $M_R$  fulfill  $[M_R]_{11} = 0$ ,  $[M_R]_{12} = O(R)$ . (In the language of generation numbers: there is no  $\Delta_R$  multiplet with  $G = 4$ , while there *is* one with  $G = 3$ ).

We denote the vev of the  $G = 3$  Higgs by  $v_3$ . If there is also a Higgs with  $G = 0$  then

$$M_R \sim \begin{pmatrix} 0 & v_3 & 0 \\ v_3 & 0 & 0 \\ 0 & 0 & v_0 \end{pmatrix}. \quad (\text{B.2})$$

We are assuming that there are no  $\Delta_R$  fields with  $G = 1, 2$ . With  $m_D$  of the form (B.1) we get

$$\frac{m(\nu_\tau)}{m(\nu_\mu)} \sim \frac{[m(\tau)]^2}{[m(e)]^{1/2}[m(\mu)]^{3/2}} \cdot \frac{v_3}{v_0} \sim \frac{v_3}{v_0} \cdot 4 \times 10^3. \quad (\text{B.3})$$

If  $v_3 \sim v_0$ , this is equivalent to  $m(\nu_\tau)/m(\nu_\mu) \sim [m(\tau)/m(\mu)]^3$ . It is interesting to note that a single  $\Delta_R$  with  $G = 0$  (and  $G_{e,\mu,\tau} = +1, -1, 0$ ) could give the same result, but its Yukawa couplings should have an opposite hierarchy to those of the Higgs doublet  $\phi$  with  $G = 0$ .

Another possibility is to have the additional Higgs field with  $G = 2$ , but no  $G = 0, 1$  fields:

$$M_R \sim \begin{pmatrix} 0 & v_3 & v_2 \\ v_3 & v_2 & 0 \\ v_2 & 0 & 0 \end{pmatrix}. \quad (\text{B.4})$$

With  $m_D$  in the Fritzsch form we get the same dependence as in (B.3), only with a factor  $[v_3/v_2]^3$ . The above two cases are based on patterns which are especially concocted for the purpose of producing  $p \geq 3$  examples. We have found no other such cases with three eigenvalues of  $O(R)$ .

If  $m(\nu_\tau)$  is at the MeV range, it is comparable to the electron mass. It is then possible that only the  $\nu_e$  and  $\nu_\mu$  masses are derived from a see-saw mechanism and the  $\nu_\tau$  mass is mostly a Dirac mass. In that case we may consider  $M_R$  matrices with only two eigenvalues of  $O(R)$ . To be specific, assume the Fritzsch form (B.1) with the appropriate generation-numbers, and that there is only a single  $\Delta_R$  multiplet, with  $G = 1$ :

$$M_R = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & R \\ 0 & R & 0 \end{pmatrix}. \quad (\text{B.5})$$

The light neutrino masses are:

$$m(\nu_\tau) = O\{[m(e)m(\mu)]^{1/2}\},$$

$$m(\nu_\mu) = O\left\{\frac{[m(\tau)]^{3/2}[m(\mu)]^{1/2}}{R}\right\}. \quad (\text{B.6})$$

The mass of  $\nu_\tau$  is a few MeV, and that of  $\nu_\mu$  can be made arbitrarily small by taking a higher scale for  $R$ . In order to obtain  $m(\nu_\mu) \leq 65$  eV we must have  $R \geq 12$  PeV.

The peculiar scenarios described in this appendix are the simplest examples we could find for see-saw matrices which evade the “reasonable see-saw” hypothesis. Their extreme ad-hoc nature strengthens our belief in the validity of that hypothesis.

### References

- [1] M. Fritsch et al., Phys. Lett. 173B (1986) 485
- [2] R. Abela et al., Phys. Lett. 146B (1984) 431
- [3] H. Albrecht et al., Phys. Lett. 163B (1985) 404
- [4] T. Yanagida, in Proc. Workshop on Unified theory and baryon number in the universe, eds. O. Sawada and A. Sugamoto, (KEK, 1979);  
M. Gell-Mann, P. Ramond and R. Slansky, in Supergravity, eds. P. van Nieuwenhuizen and D. Freedman (North-Holland, 1980)
- [5] H. Harari and Y. Nir, Phys. Lett. 188B (1987) 163
- [6] S.S. Gerstein and Ya.B. Zeldovich, JETP Lett. 4 (1966) 120;  
R. Cowsik and J. McClelland, Phys. Rev. Lett. 29 (1972) 669
- [7] B.W. Lee and S. Weinberg, Phys. Rev. Lett. 39 (1977) 165
- [8] D.A. Dicus, E.W. Kolb and V.L. Teplitz, Phys. Rev. Lett. 39 (1977) 168; (E) 39 (1977) 973
- [9] J. Bernstein and G. Feinberg, Phys. Lett. 101B (1981) 39;  
A.S. Szalay and G. Marx, Astron. Astrophys. 49 (1976) 437
- [10] Y. Chikashige, R.N. Mohapatra and R.D. Peccei, Phys. Rev. Lett. 45 (1981) 1926
- [11] G.A. Tammann, A. Sandage and A. Yahil, in Physical cosmology, eds. R. Balian, J. Audouze and D.N. Schramm (North-Holland, 1980)
- [12] E.D. Loh and E.J. Spillar, Astrophys. J. 307 (1986) L1
- [13] S. Weinberg, in Gravitation and cosmology (Wiley, 1972)
- [14] P.B. Pal, Nucl. Phys. B227 (1983) 237
- [15] J. Bernstein, L.S. Brown and G. Feinberg, Phys. Rev. D32 (1985) 3261
- [16] E.W. Kolb and K.A. Olive, Phys. Rev. D33 (1986) 1202, (E) D34 (1986) 2531
- [17] K. Sato and M. Kobayashi, Prog. Theor. Phys. 58 (1977) 1775
- [18] R. Cowsik, Phys. Rev. Lett. 39 (1977) 784;  
D.A. Dicus, E.W. Kolb and V.L. Teplitz, Astrophys. J. 221 (1978) 327
- [19] D.A. Dicus, E.W. Kolb, V.L. Teplitz and R.V. Wagoner, Phys. Rev. D17 (1978) 1529;  
S. Miyama and K. Sato, Prog. Theor. Phys. 60 (1978) 1703;  
E.W. Kolb and R.J. Scherrer, Phys. Rev. D25 (1982) 1481
- [20] A. Dar, J. Goodman and S. Nussinov, Princeton preprint PRINT-86-1445 (1986)
- [21] D. Lindley, Astrophys. J. 294 (1985) 1
- [22] L.M. Krauss, Harvard preprint HUTP-85/A040 (1985);  
S. Sarkar and A.M. Cooper, Phys. Lett. 148B (1984) 347
- [23] A.D. Dolgov and Ya.B. Zeldovich, Rev. Mod. Phys. 53 (1981) 1
- [24] Y. Hosotani, Nucl. Phys. B191 (1981) 411

- [25] R.N. Mohapatra and G. Senjanović, *Phys. Rev. Lett.* **44** (1980) 912; *Phys. Rev. D* **23** (1981) 165
- [26] M. Magg and C. Wetterich, *Phys. Lett.* **94B** (1980) 61
- [27] J. Schechter and J.W.F. Valle, *Phys. Rev. D* **25** (1982) 774
- [28] L.A. Ahrens et al., *Phys. Rev. D* **31** (1985) 2732
- [29] N. Ushida et al., *Phys. Rev. Lett.* **57** (1986) 1694
- [30] D.A. Bryman et al., *Phys. Rev. Lett.* **50** (1983) 1546;  
G. Bernardi et al., *Phys. Lett.* **166B** (1986) 479
- [31] P.B. Pal and L. Wolfenstein, *Phys. Rev. D* **25** (1982) 766
- [32] J.F. Niéves, *Phys. Rev. D* **28** (1983) 1664
- [33] E. Witten, *Phys. Lett.* **91B** (1980) 81
- [34] M. Roncadelli and G. Senjanović, *Phys. Lett.* **107B** (1981) 59
- [35] SINDRUM collaboration, *Nucl. Phys. B* **260** (1985) 1
- [36] ARGUS collaboration, *Phys. Lett.* **185B** (1987) 228
- [37] U. Chattopadhyay and P.B. Pal, *Phys. Rev. D* **34** (1986) 3444
- [38] W. Buchmüller and D. Wyler, *Nucl. Phys. B* **268** (1986) 621
- [39] G.L. Kane and W.B. Rolnick, *Nucl. Phys. B* **232** (1984) 21
- [40] D. Ware and M.E. Machacek, *Phys. Lett.* **147B** (1984) 415;  
C. Hearty et al., *Phys. Rev. Lett.* **58** (1987) 1711
- [41] R. Barbieri, C. Bouchiat, A. Georges and P. Le Doussal, *Phys. Lett.* **156B** (1985) 348; *Nucl. Phys. B* **269** (1986) 253
- [42] M. Fukugita, S. Watamura and M. Yoshimura, *Phys. Rev. Lett.* **48** (1982) 1522
- [43] D.S.P. Dearborn, D.N. Schramm and G. Steigman, *Phys. Rev. Lett.* **56** (1986) 26
- [44] A. Jodidio et al., *Phys. Rev. D* **34** (1986) 1967
- [45] G.B. Gelmini and M. Roncadelli, *Phys. Lett.* **99B** (1981) 411
- [46] F. Wilczek, *Phys. Rev. Lett.* **49** (1982) 1549
- [47] S.L. Glashow, *Phys. Lett.* **187B** (1987) 367
- [48] G.B. Gelmini and J.W.F. Valle, *Phys. Lett.* **142B** (1984) 181
- [49] B. Grinstein, J. Preskill and M.B. Wise, *Phys. Lett.* **159B** (1985) 57
- [50] A. Kumar and R.N. Mohapatra, *Phys. Lett.* **150B** (1985) 191
- [51] S.L. Glashow, in *Fifth Workshop on Grand Unification*, eds. K. Kang, H. Fried and P. Frampton (World Scientific, Singapore, 1984)
- [52] H. Georgi and D.V. Nanopoulos, *Nucl. Phys. B* **155** (1979) 52;  
R. Barbieri, D.V. Nanopoulos, G. Morchio and F. Strocchi, *Phys. Lett.* **90B** (1980) 91
- [53] E. Witten, *Nucl. Phys. B* **268** (1986) 79
- [54] H. Fritzsch, *Phys. Lett.* **73B** (1978) 317; *Nucl. Phys. B* **155** (1979) 189
- [55] UA1 collaboration, *Phys. Lett.* **185B** (1987) 241
- [56] W. Marciano, in *Proc. 23rd Int. Conf. on High Energy Physics*, Berkeley, CA (1986), to be published
- [57] G. Beall, M. Bander and A. Soni, *Phys. Rev. Lett.* **48** (1982) 848
- [58] G. Ecker, W. Grimus and H. Neufeld, *Phys. Lett.* **127B** (1983) 365;  
M. Leurer, *Nucl. Phys. B* **266** (1986) 147
- [59] H. Harari and M. Leurer, *Nucl. Phys. B* **233** (1984) 221
- [60] R.N. Cahn and H. Harari, *Nucl. Phys. B* **176** (1980) 135
- [61] L. Wolfenstein, *Phys. Rev. D* **17** (1978) 2369;  
S.P. Mikheyev and A.Yu. Smirnov, in *Proc. 10th Int. Workshop on Weak Interactions*, Savonlinna, Finland (1985);  
H.A. Bethe, *Phys. Rev. Lett.* **56** (1986) 1305
- [62] C.S. Lim and T. Inami, *Prog. Theor. Phys.* **67** (1982) 1569
- [63] R.N. Mohapatra and P.B. Pal, *Phys. Lett.* **179B** (1986) 105