A DYNAMICAL THEORY FOR THE RISHON MODEL [☆]

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We propose a composite model for quarks and leptons based on an exact $SU(3)_C \times SU(3)_H$ gauge theory and two fundamental $J = 1/2$ fermions: a charged T-rishon and a neutral V-rishon. Quarks, leptons and W-bosons are $SU(3)_H$-singlet composites of rishons. A dynamically broken effective $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge theory emerges at the composite level. The theory is "natural", anomaly free, has no fundamental scalar particles, and describes at least three generations of quarks and leptons.

The rishon model has been proposed [1,2] as a simple "counting scheme" describing quarks and leptons as composite objects. There are only two fundamental $J = 1/2$ building blocks: a charged T-rishon and a neutral V-rishon. Quarks and leptons are composites of three rishons (rrr) or three antirishons ($\bar{r}\bar{r}\bar{r}$). The model is extremely economical and is an attractive "mnemonic" for reproducing the correct spectrum of quarks and leptons of *one* generation. However, the original scheme was not supplemented by a dynamical theory, and, thus, a large number of critical questions remained unanswered: What binds the rishons? How do rishons form (almost) massless composites? How does one construct several generations of quarks and leptons with mass splittings which are much smaller than the inverse size of the composite system? Why are there no spin 3/2 quarks and leptons? Why do we observe only composites of rrr or $\bar{r}\bar{r}\bar{r}$ and not, for instance, rr$\bar{r}$? What kind of statistics is obeyed by the rishons? How does one produce the electroweak gauge group in the composite level?

In studying various possible dynamical schemes for the rishon model we have reached the conclusion that two extreme alternatives are open to us. The "radical" view [1–3] would insist on only two very heavy rishons obeying, at short distances, new "unusual" rules such as

unconventional statistics, nonlocal field theory, nonassociative algebra and the like. The "conventional" view would adopt the notion [4] that the (almost) massless composites are constructed from massless rishons which must therefore be confined (or else would be observed). This would lead us to a color-type symmetry at the rishon level, sacrificing some of the economy of the original scheme.

While one should never exclude "radical" possibilities, we are tempted to pursue to the limit the conventional ideas of local quantum field theory with local gauge invariance. In this paper we propose a "conventional" dynamical theory for the rishon model. The theory has several attractive features. In particular, it attempts to answer all the questions listed above. Some of its ingredients require further investigation. The price we must pay is greater complexity in the underlying rishon level.

We start with a local gauge theory based on the gauge group $SU(3)_C \times SU(3)_H$ (C = color, H = hypercolor). This is assumed to be an *exact symmetry of nature at all levels*, and we will not break it at any stage. We assume that the two coupling constants g_C, g_H are unequal (say, $g_C < g_H$). If Λ_H is the scale in which $g_H \approx 1$, we have $g_C(\Lambda_H) < 1$. The two scale parameters then obey $\Lambda_C \ll \Lambda_H$. All of matter is constructed from two types of $J = 1/2$ *massless* rishons, obeying *ordinary Fermi* statistics: the T-rishon has $Q = +1/3$ and transforms like a (3, 3) representation of $SU(3)_C \times SU(3)_H$.

The V-rishon has $Q = 0$ and is assigned to $(\bar{3}, 3)$. The $\bar{V}$ and $\bar{T}$ are, respectively, in $(3, \bar{3})$ and $(\bar{3}, \bar{3})$. The fundamental lagrangian includes the couplings of rishons to gluons and to hypergluons, as well as gluon and hypergluon self-couplings, but no mass terms. It conserves n_T and n_V (the net numbers of T's and V's). It therefore possesses a global $U(1) \times U(1)$ symmetry. The two $U(1)$ quantum numbers can be chosen in several ways. We find it convenient to define

$$R = \tfrac{1}{3}(n_T + n_V) \qquad \text{(rishon number)} ,$$

$$\Upsilon = \tfrac{1}{3}(n_T - n_V) = B - L \quad \text{(baryon minus lepton number)}.$$

The electric charge is simply given by

$$Q = \tfrac{1}{2}(R + \Upsilon) .$$

At this stage, we leave open the possibility of assuming that $U(1)$ of the electric charge is a local gauge symmetry rather than a global symmetry.

All the rishons and antirishons have different $SU(3) \times SU(3)$ assignments. Hence, the global "flavor" symmetry at the rishon level is $U(1)_R \times U(1)_{B-L}$ and not any $SU(N)$. Since there are no masses in the fundamental lagrangian, we actually have a chiral $[U(1)]^4$ symmetry with two axial $U(1)$ factors. The divergence of one axial $U(1)$ factor is proportional to a combination of $(F\bar{F})_C$ and $(F\bar{F})_H$. The other axial $U(1)$ leads to a conserved axial charge [5].

At energies well above Λ_H we find asymptotically free rishons. As we decrease the energy towards Λ_H, all hypercolor nonsinglets presumably become confined. Only composites transforming like $SU(3)_H$-singlets may appear as free particles at energies below Λ_H.

What can we say about the masses of $SU(3)_H$-singlet composites? They may be of order Λ_H, or they may be small with respect to Λ_H. As long as we do not know how to calculate the confinement mechanism, we cannot determine which hypercolor singlets are (almost) massless and which are heavy. However, we require that, if some composites have small masses, their effective low energy theory must be "natural", namely, the *effective* lagrangian of the small-mass composites is renormalizable (and anomaly free).

These conditions are not sufficient in order to guarantee that the effective low energy lagrangian actually follows from the fundamental lagrangian. However, the conditions are necessary, suggesting that a theory which obeys them is a candidate for the correct theory.

What we propose to do here is to find a simple set of almost massless $SU(3)_H$-singlet composites which obeys the above requirements of a natural theory. We will then show that no simple solution exists, thus strengthening the probability that our theory is correct. However, we can neither prove uniqueness nor can we show that the confinement mechanism actually produces massless composites for the relevant specific states.

The simplest composite *fermions* consist of three constituents. A list of all possible combinations of three rishons and antirishons is given in table 1. At energies Λ_H, the states in the two central columns are confined "hyperleptons" and "hyperquarks". Their effective masses are of order Λ_H. *We therefore understand why only three-rishon and three-antirishon states are "observed"* while $rr\bar{r}$ and $r\bar{r}\bar{r}$ states are not. The remaining two columns represent hypercolor singlets forming color-singlet leptons, color-triplet quarks and, in principle, also higher color multiplets. *We now conjecture that for each configuration, the smallest color multiplet is approximately massless on the scale of Λ_H.* As stated above, we will show that this conjecture provides us with a "natural" theory, a condition which is not shared by any other simple set of massless composites.

Table 1
Composites of three rishons and antirishons. Each row or column is labeled by the smallest allowed multiplet of the relevant SU(3) group.

$SU(3)_C, B - L$	$SU(3)_H, R$			
	$1, 1$	$3, 1/3$	$\bar{3}, -1/3$	$1, -1$
$1, 1$	e^+	hyperleptons		$\bar{\nu}_e$
	TTT	$TT\bar{V}$	$T\bar{V}\bar{V}$	$\bar{V}\bar{V}\bar{V}$
$3, 1/3$	u	hyperquarks		d
	TTV	$TT\bar{T}$	$\bar{V}T\bar{T}$	$\bar{V}\bar{V}\bar{T}$
		$T\bar{V}\bar{V}$	$\bar{V}\bar{V}\bar{V}$	
$\bar{3}, -1/3$	$\bar{d}$			$\bar{u}$
	TVV	$VT\bar{T}$	$\bar{T}\bar{T}\bar{T}$	$\bar{V}TT$
		$V\bar{V}\bar{V}$	$\bar{T}\bar{V}\bar{V}$	
$1, -1$	ν_e	hyperleptons		e^-
	VVV	$V\bar{V}\bar{T}$	$\bar{V}T\bar{T}$	$\bar{T}\bar{T}\bar{T}$

It is clear that a TTT-singlet, TTV-triplet, TVV-antitriplet, VVV-singlet and their antiparticles precisely reproduce the quantum numbers of one generation of quarks and leptons [1,2].

The typical radius of such composite fermions will be of order Λ_H^{-1}, the range of the confining hypercolor potential. Thus we have composite *light* fermions with *tiny radii*, as required by the limits on pointlike leptons and quarks [1]. Let us first consider the singlet TTT. It is fully antisymmetric under both color and hypercolor. To obey overall Fermi statistics it must therefore be totally antisymmetric under the Lorentz group. A massless, left- and right-handed rishon transforms according to the $[(1/2, 0) + (0, 1/2)]$ representation of the Lorentz group. If we ignore derivative couplings, the totally antisymmetric TTT-state must therefore transform like $(0, 1/2) + (1/2, 0)$, thus creating precisely one $J = 1/2$ composite positron. *Not only do rishons obey ordinary statistics, but we have no room for a $J = 3/2$ lepton!* The required $J = 1/2$ quark states are also perfectly consistent with Fermi statistics, but $J = 3/2$ states could exist. They may be excluded on the basis of general arguments against $J = 3/2$ massless particles [4,6] or they may be heavy.

So far we have not discussed the weak interactions. Are they already included in the theory? Our hypercolorless leptons contain hypercolored rishons inside a radius of order Λ_H^{-1}. Two leptons will interact with each other by short-range residual hypercolor forces. These are analogous to the hadronic forces among two colorless hadrons containing quarks. We propose that this residual hypercolor force is, indeed, the weak interaction. But does it have the observed properties of the weak interactions?

At energies below Λ_H only $SU(3)_H$-singlet composites will appear in the effective low-energy lagrangian. Since the original symmetry is $SU(3)_C \times SU(3)_H \times U(1)_R \times U(1)_{B-L} \times U(1)_{axial}$, the *effective* lagrangian would appear to have only an $SU(3)_C \times U(1)_R \times U(1)_{B-L} \times U(1)_{axial}$ symmetry. However, at the composite level, new symmetries may arise. For instance, every $R = 1$ composite fermion in table 1 (say, TTT) has precisely the same interactions as the corresponding $R = -1$ fermion with the same $(B - L)$ value ($\overline{V}\overline{V}\overline{V}$, in our example). We may define an operator transforming $e^+ \to \bar{\nu}_e$, $u \to d$, $\bar{d} \to \bar{u}$, $\nu_e \to e^-$ and a second operator performing the opposite transformation. These two operators, together with $\frac{1}{2}R$, form an SU(2)

algebra, under which our effective lagrangian is *globally* invariant. In the limit of massless composite fermions, the symmetry is larger and we have an $SU(2)_L \times SU(2)_R$ global symmetry, with $\frac{1}{2}R = I_L^3 + I_R^3$, and the axial U(1) charge being $I_L^3 - I_R^3$. *The overall symmetry of the effective lagrangian is, therefore, $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$,* precisely the group of the left–right symmetric extension [7] of the "standard model".

The operators transforming $R = -1$ fermions into $R = 1$ fermions correspond to the quantum numbers of TTTVVV composites. Such composites may exist and may be $SU(3)_C \times SU(3)_H$ singlets. We may identify them as the W^+-bosons (W_L^+, W_R^+). Similarly, we identify W^- and W^3, respectively, as composites of $\overline{T}\overline{T}\overline{T}VVV$ and of a combination of $T\overline{T}$ and $V\overline{V}$ pairs. If such composites have masses well below Λ_H and if their relative couplings correspond to the usual gauge theory couplings, the effective lagrangian will be locally gauge invariant under $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, at energies below Λ_H. Above Λ_H, the $SU(2)_L \times SU(2)_R$ symmetry is lost and the rishons themselves do not have well-defined transformation properties under it. If the vector boson couplings differ from those of the local gauge theory, the effective lagrangian is not renormalizable and the theory is not "natural". We therefore *conjecture* that, at the low energy scale, the lagrangian possesses all the ingredients of a local gauge theory. We can offer some plausibility arguments (but no proof) indicating that the vector boson couplings are, indeed, the correct ones.

Our low energy fermion spectrum contains one pair of leptons (e^-, ν_e) accompanied by one pair of quarks (u, d) and is therefore free from $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ anomalies. It is at this point that we find that no other simple solution is allowed. If we ignore derivative couplings, i.e. allow only three-rishon composites in Lorentz group representations contained in the product of three $[(1/2, 0) + (0, 1/2)]$ multiplets, *there is no other set of massless $SU(3)_H$-singlet composites that is anomaly-free in $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$.*

The electroweak angle θ_w can be uniquely determined in our theory and we find $\sin^2\theta_w = 0.25$, in very good agreement with experiment. This has been shown in the original rishon model paper [1], but we repeat the main point. Since $Q = \frac{1}{2}[R + (B - L)]$, we may write:

$$J_Q = 2^{-1/2}(J_R + J_{B-L}) \, ,$$

where the three currents are properly normalized $r\bar{r}$ currents coupled to the appropriate U(1) charges. The standard definition of θ_w corresponds to:

$$J_Q = \sin\theta_w (J_{3L} + J_{3R}) + (\cos 2\theta_w)^{1/2} J_{B-L} \, .$$

A trivial comparison gives: $\sin^2\theta_w = \frac{1}{4}$. *This prediction is not expected to change considerably between* Λ_H *and present energies.*

What can we say about the mass of the W-bosons? Here the situation is complicated, but extremely interesting. In the first stage, we would like to suggest that the W-bosons are massless. We can only *conjecture* that the massless $W^{\pm}$ form as *vector* condensates [+1] of six rishons. The question of vector condensates is not well understood and we will not discuss it any further here. We do have, however, Higgs-like mesons. Our leptons and quarks can bind due to the residual hypercolor forces, forming scalar condensates and providing mass terms to the W-bosons and the fermions. This is somewhat similar to the "technicolor" mechanism [9]. The important difference between our picture and the usual one [9] is that our Higgs particles are composites of leptons (or quarks) and all necessary masses are produced without any need of a further hierarchy of mass generating mechanisms. Our composite Higgs particles have precisely the necessary properties for producing the correct mass pattern of the gauge bosons [10]. Note that all along the way, while we have produced an *effective* dynamically broken $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ gauge theory, we did not break our original $SU(3)_C \times SU(3)_H$ symmetry in any way.

How do we form several generations of quarks and leptons? The same reasoning that led us to assume that certain three-rishon composites are approximately massless, would tell us that some five-rishon $SU(3)_H$-singlet composites may also be massless in a "natural" way. These $SU(3)_H$ singlets consist of $(4r + \bar{r})$ or $(4\bar{r} + r)$. In general, we could have states with $B - L = 5/3, 1, 1/3, -1/3, -1, -5/3$. These include the usual $B - L = \pm 1, \pm 1/3$ quantum numbers of ordinary quarks and leptons but also a peculiar $B - L = 5/3$ doublet of fermions with $Q = 4/3, 1/3$. The complete list of potential five-rishon composites would therefore, in general, in-

clude a_1 doublets of ordinary leptons, a_2 doublets of ordinary tricolored quarks and a_3 doublets of tricolored peculiar $B - L = 5/3$ objects. The anomaly condition for $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ is:

$$a_1 = a_2 + 5a_3 \, .$$

But the total number of lepton pairs which we can form from five rishons is smaller than five, and $a_2 a_3$ must be non-negative integers. Hence, the only solution is $a_1 = a_2, a_3 = 0$, namely, a normal doublet of tricolored quarks for each normal doublet of leptons, and no peculiar objects. We therefore conclude that, *in a "natural" theory, three-rishon and five-rishon states can form at least three identical generations of massless quarks and leptons with precisely the observed spectrum and no "exotics".* We cannot exclude a variety of exotic fermions consisting of seven or more rishons. But when these acquire masses, they will presumably be heavier. All our massless fermions obtain masses via the same mechanisms, and nothing prevents the mixing of three-rishon and five-rishon states. Hence, in general, we should definitely have Cabibbo mixing and a complicated fermion mass matrix. The calculation of fermion masses and Cabibbo angles should, in principle, be performed by computing the relevant color and hypercolor interactions. It is, however, crucial that the quark and lepton masses in all three generations are small compared with their inverse effective radii.

Let us summarize: we propose a gauge theory of massless rishons based on $SU(3)_C \times SU(3)_H$. Without ever breaking the *original* symmetry we find, at the composite level, an effective $SU(3)_C \times SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ dynamically broken gauge theory with the correct spectrum of quarks and leptons appearing in several identical generations. The theory has no fundamental scalar particles, no anomalies, and it is completely natural and renormalizable at all levels. A dynamical symmetry breaking mechanism appears automatically, without invoking any new assumptions of new fundamental particles. Our theory is not consistent with the ideas of "grand unification theories". Our electroweak group simply does not exist at 10^{15} GeV. Its couplings are determined by the hypercolor couplings, but they are not given in terms of Clebsch–Gordan coefficients.

Finally, we wish to enumerate some of the many important areas of investigation which require further analysis:

[+1] For a recent discussion of vector condensates, with earlier references, see e.g. Bais and Frere [8].

(i) A proper understanding of the masses and couplings of the W-bosons.

(ii) A detailed analysis of the various mass generating mechanisms [10].

(iii) An analysis of the way in which parity is violated within the left–right symmetric $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [10].

(iv) An understanding of the fermion generation mixing, mass matrix, and the neutrino mass problem [10].

(v) A study of the possibility that confinement leads, under certain conditions, to massless composites.

(vi) A phenomenological discussion of baryon non-conservation, hyperhadrons, right-handed W-bosons, etc. There is certainly no "desert" in our theory, and if Λ_H is anywhere between 10^3–10^6 GeV, we may be able to detect its indirect effects within the next decade.

We hope to return to these and other aspects of the theory in future publications.

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