

## RESONANCES—Theory

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### 1. INTRODUCTION

We have heard from the previous three rapporteurs<sup>1-3)</sup> that our experimental colleagues have reached remarkable achievements in their search for resonances. This point is perhaps best illustrated by reproducing the entire information on resonances included in the first edition<sup>4)</sup> (dated 20 March, 1958) of the now famous "Rosenfeld Tables". Using the same information density with which the August 1968 table fills two-and-a-half condensed pages<sup>5)</sup>, the 1958 version is shown in Fig. 1. We

(2.2, 2/2) $\rho$ Resonance
Center-of-mass momentum: $P_{cm} = 230 \text{ Mev}/c$
Lab-system momentum: $P_{lab} = 302 \text{ Mev}/c$ ( $T_p = 194 \text{ Mev}$ )

Fig. 1 The entire information on resonances included in the 1958 "Rosenfeld Table".

theorists cannot claim similar progress in understanding the observed resonance spectrum. In fact, at the time of the last conference, we thought that all we had to do was to explain why certain resonances existed and why they possessed their specific properties. As you will see, now we are not even sure any more that we know what a resonance is.

In this report I propose to discuss two main theoretical topics related to the interpretation of resonant states: i) the present status of various hadron classification schemes; and, ii) the role played by resonance states in our understanding of strong interaction dynamics.

The material of this report is organized as follows:

Section 2 deals with various theoretical models for meson spectroscopy including the quark model, Regge theory, SU(3), and chiral SU(2)  $\times$  SU(2).

Section 3 treats the baryon spectrum with an emphasis on SU(3) and the quark model.

Section 4 is devoted to the general question of the relation between t-channel Regge trajectories and s-channel resonances. We discuss the possible mechanisms which may lead to loops in the Argand diagram for partial wave amplitudes. At the end of this discussion we return to the meson and baryon spectrum, and try to find to what extent the qualitative predictions of the t-channel Regge picture are reflected by the experimentally observed states.

In Section 5 we briefly discuss the speculative picture according to which most of the observed hadron reactions may be described in terms of sums of resonance excitations.

Section 6 is a short summary of our main discussion points.

Our choice of material as well as our list of references are certainly incomplete. Many of the interesting contributions submitted to this Conference are not discussed and many others are discussed only briefly, mainly because of time limitations.

This written report includes some material (Sections 2.5, 3.4, 4.4, 5.2, 5.3) which was not presented in the actual talk, but which it was promised would be included in the Proceedings.

### 2. MESON CLASSIFICATION SCHEMES

#### 2.1 The "old-fashioned" quark model

The usual quark-model description of meson states<sup>6)</sup>, temporarily ignoring "radial" excitations, includes the "energy levels" of Fig. 2, in which we have limit-

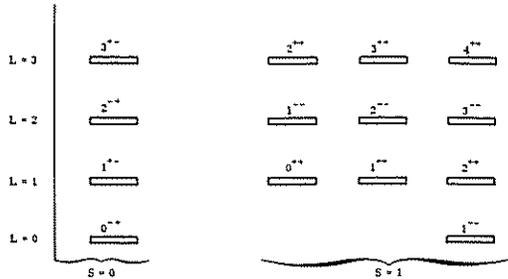


Fig. 2  $L \leq 3$   $q\bar{q}$  "energy levels". Every level is denoted by  $J^{PC}$  and represents an SU(3) nonet. L-S coupling, spin dependence, and SU(3) breaking effects are ignored.

ed ourselves to orbital angular momentum  $L \leq 3$  and ignored spin dependence, SU(3) breaking, and L-S splittings<sup>7)</sup>. Every "level" presumably corresponds to a full nonet, including four different isomultiplets:  $I = 1, 1/2, 0, 0$ . The present status<sup>1)</sup> of meson states below 1.5-1.6 GeV leads to the assignments of Table 1.

TABLE 1

$L = 0$  and  $L = 1$  quark-antiquark meson states

L	S	$J^{PC}$	$I = 1$	$I = 1/2$	$I = 0$	$I = 0$
0	0	$0^{++}$	$\pi$	K	$\eta$	$\chi^0$
	1	$1^{+-}$	$\rho$	$K^*(890)$	$\omega$	$\phi$
1	1	$0^{++}$	$\delta$ (960)	$K\pi$ (1100)	$\sigma$ (750)	$S^*(1070)$
	1	$1^{++}$	$A_1$ (1070)	$K^*$ (1230)	D (1280)	?
	1	$2^{++}$	$A_2$ (1310)	$K^*$ (1420)	$f^0$ (1260)	$f^*(1515)$
	0	$1^{+-}$	B (1220)	$K^*$ (1320)	?	?

The following remarks should be made with respect to the particle states of Table 1:

- a) In the pseudoscalar nonet, an octet-singlet mixing angle of the order of  $10^\circ$ - $20^\circ$  seems to be necessary in order to explain a large number of phenomena. The quark model may give hints for the sign of this angle (see Sections 2.3.1, 2.3.4, and 2.4), but no conclusive determination is possible at present.
- b) The  $\delta$ (960) has now been seen in  $K\bar{p}$  and  $\bar{p}p$  reactions<sup>8,9)</sup> and it decays, at least part of the time, to  $\eta\pi$ . We may now consider it as a fairly established member of the scalar nonet.
- c)  $K\pi$  S-wave phase shifts obtained from extrapolation to the pion pole in KN reactions dominated by  $\pi$  exchange indicate a  $0^+$  resonance at 1100-1200 MeV<sup>10)</sup>. Assuming that  $\kappa$ (730) is dead<sup>1)</sup>, the 1100 MeV state should be the strange member of the scalar nonet.

d) The  $\sigma$ ( $J^{PC} = 0^{++}, I^G = 0^+$ ) may have any mass between 700-1000 MeV (probably around 750 MeV), and its existence is agreed upon by all  $\pi\pi$  "phase-shifters"<sup>11)</sup>. It has not yet been definitely observed in a direct experiment. It is probably an almost pure "non-strange"  $q\bar{q}$  state.

e) The  $S^*(1070)$  is probably the  $\lambda\bar{\lambda}$  state ( $\lambda$  = strange quark) of the scalar nonet. There are possible indications of a  $\pi\pi$  state around 1050, which may or may not<sup>12)</sup> be the same state. Assuming  $S^* \neq \pi\pi$ , the  $\lambda\bar{\lambda}$  interpretation is the most natural one, leading to the "universal" arc  $\cos \sqrt{2/3}$  octet-singlet mixing angle for the  $0^+$  nonet.

f) In spite of the experimental doubts<sup>1)</sup> we continue to believe that the  $A_1$  exists and is a  $1^{++}$  state (see also Section 4.4.4).

g) The  $K^*(1230)$  may be the same as the C meson, and may or may not be a  $1^+$  state. We tentatively assign this "lower half of the Q-bump" to the  $A_1$  nonet, recalling that the two axial  $K^*$ 's can mix<sup>13)</sup>.

h) Since D(1280) is heavier than  $A_1$  and  $K^*(1230)$ , our first guess would be to associate it with the  $\lambda\bar{\lambda}$   $J^{PC} = 1^{++}$  state, whilst the other  $1^{++}$  isoscalar should be found around the  $A_1$  mass. The recently observed<sup>9)</sup> decay  $D \rightarrow \delta + \pi$  indicates, however, that D has at least some non-strange quark components.

i) Any educated guess would place the missing  $1^{++}$  isoscalar around 1050-1300 MeV. It could decay to  $4\pi, K\bar{K}\pi, KK^*, \pi\pi\eta, \pi\delta$ , etc. The absence of this state should probably start to embarrass us. If its mass is below 1100 MeV, only the  $4\pi$  and  $\pi\pi\eta$  modes are allowed, but in any case we should have probably seen it by now.

j) We assume that at least one of the  $A_2$  states (if there are more than one) is a  $2^{++}$ .

k)  $K^*(1320)$  is supposedly the other  $1^+K^*$ , corresponding to the high-mass portion of the Q-bump.

l) The untimely death<sup>1,14)</sup> of the H meson at 990 MeV has left us with two missing  $1^{+-}$  isoscalars.

Their masses could typically be 1200 and 1400 MeV, if they are a pure "non-strange" and a pure "strange"  $q\bar{q}$  state, respectively. At least one of them should decay to  $\rho\pi$ , but if its mass is around 1200 MeV, I do not see how we could have possibly isolated it, in view of the  $A_1$ ,  $A_{1.5}$ ,  $A_2^L$ , and  $A_2^H$  confusion. The "cleanest" way of seeing this state is in the  $\rho^0\pi^0$  invariant mass plot, which should not show the  $I = 1$  states. Both missing  $I = 0$ ,  $J^{PC} = 1^{+-}$  states should decay to  $K\bar{K}\pi$ , and in particular to  $K_1^0 K_2^0 \pi^0$ , a mode which distinguishes them from the D and the E mesons.

m) Among the other mesons below 1.5 GeV we should mention E(1410) as a possible radial excitation of  $\eta$  in the  $0^-$  nonet. We do not know where the other eight states of this radially excited pseudoscalar nonet are or why we do not see a "radially excited"  $\pi$ , which should be diffractively produced in  $\pi p$  collisions. The only candidate for such an excited  $\pi$ , at present, could be  $A_{1.5}(1170)$ , if it exists.

n)  $A_2^L(1270)$ , a third  $K^*(1280?)$  in the Q-region, a possible D-wave  $\pi\pi$  state at 1050, a possible  $K\pi$  resonance at 1260, and a few other hints here and there<sup>1)</sup>, are among the possible future sources of difficulty to the simple "old-fashioned" quark model.

The situation concerning  $L > 1$ ,  $m > 1500$  MeV mesons is sufficiently confused<sup>1)</sup> to prevent us from making any serious attempts at classification in this region. We would like to emphasize, however, that even the usual quark model (with radial excitations) predicts many additional states between 1600-1900 MeV, which have yet to be discovered.

## 2.2 Quarks, daughter trajectories, and the new Gell-Mann-Zweig model

A brief glance at Fig. 2 immediately reminds us of one severe restriction imposed by the ordinary quark model on the allowed meson quantum numbers. All natural parity mesons ( $0^+$ ,  $1^-$ ,  $2^+$ , ...) must have "normal" charge conjugation, namely, as for all non-strange, neutral, natural parity mesons  $C = P$ . From the purely experimental point of view, this is probably one of the most striking successes of the  $q\bar{q}$  description of mesons, since we do not have a single established meson which violates this rule. On the other hand, from the purely theoretical standpoint we may be facing a difficulty. The classification of meson states accord-

ing to Regge trajectories has had some remarkable successes, such as the probable existence of the  $J^P = 3^-$  g meson on the  $\rho$  trajectory, or the R-S-T-U sequence of mesons discovered by the CERN Missing-Mass Spectrometer Group. Various theoretical arguments, including group theoretical speculations<sup>15)</sup> as well as dynamical bootstrap models<sup>16,17)</sup>, indicate that the daughter trajectories<sup>18)</sup> (which are needed at  $t = 0$  to guarantee the correct analytic properties) are actually parallel to the parent trajectories of a given sequence. If this is the case, and if the parallel daughter trajectories actually materialize into particles at some or all of the right-signature points, we find ourselves faced with a dilemma. All the odd daughters of all natural parity trajectories, as well as many other daughter states, are not allowed by the quark model. This is clearly demonstrated in Fig. 3, where the exchange-degenerate daughters of the exchange-degenerate  $\rho$ - $A_2$  trajectories are shown to be illegitimate within the framework of the "old"  $q\bar{q}$  model.

This theoretical inconsistency should worry any person who believes in the relevance of the quark model on the one hand and the sequence of parallel daughter

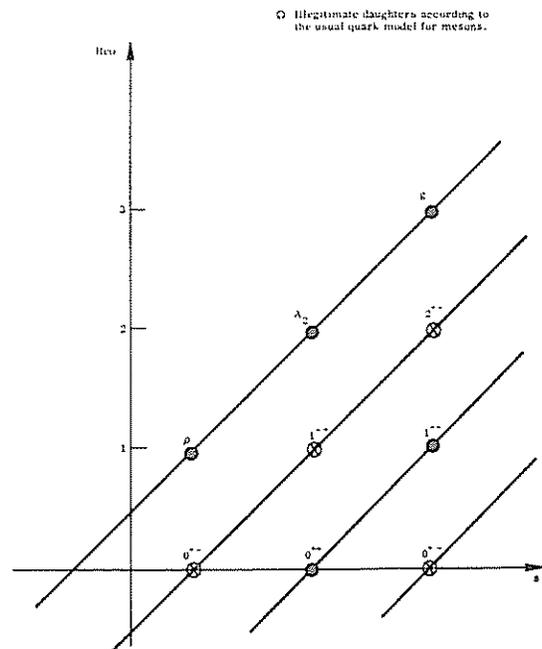


Fig. 3 The  $\rho$  and  $A_2$  trajectories (assumed to be degenerate) and their daughter-trajectory sequence. All states have natural parity but the "odd" daughters have  $C = -P$  and are therefore not allowed by the ordinary  $q\bar{q}$  picture of meson states.

trajectories on the other hand, to the observed spectrum of mesons.

A way out of this difficulty has recently been proposed by Gell-Mann and Zweig<sup>19)</sup>. They point out that the contradiction stems from the fact that the non-relativistic quark model involves states which are identified according to their O(3) properties, and which are supposed to exhibit the general characteristics of the states of a three-dimensional harmonic-oscillator type potential. On the other hand, the entire group-theoretic formulation of the daughter sequences is based on the existence of a symmetry larger than O(3) -- either O(3,1) or O(4). Any given O(3) state has to appear in conjunction with others, forming a complete O(3,1) or O(4) representation. Gell-Mann and Zweig therefore introduce a four-dimensional oscillator scheme in which both the "quark-spin" S and the "orbital angular momentum" L are O(4) quantum numbers and are accompanied by the extra quantum numbers required by a complete O(4) classification. For example, the usual spin triplet (S = 1) is accompanied by a "fourth component" of its O(4) four-vector. In the case of the lowest-lying vector mesons, this "fourth component" is interpreted as the daughter state of the 1<sup>---</sup> nonet, presumably having a similar mass and J<sup>PC</sup> = 0<sup>+-</sup>.

The meson spectrum proposed by Gell-Mann and Zweig is shown in Fig. 4. They essentially introduce a complete sequence of J ≥ 0 daughters for all natural

$\rho_{11}^{+-}$ $\rho_{10}^{+0}$ $\rho_{10}^{00}$ $\rho_{11}^{0-}$	L = 3	$\rho_{11}^{+-}$ $\rho_{10}^{+0}$ $\rho_{10}^{00}$ $\rho_{11}^{0-}$ $\rho_{11}^{+-}$ $\rho_{10}^{+0}$ $\rho_{10}^{00}$ $\rho_{11}^{0-}$ $\rho_{11}^{+-}$ $\rho_{10}^{+0}$ $\rho_{10}^{00}$ $\rho_{11}^{0-}$
$\rho_{11}^{+-}$ $\rho_{10}^{+0}$ $\rho_{10}^{00}$	L = 2	$\rho_{11}^{+-}$ $\rho_{10}^{+0}$ $\rho_{10}^{00}$ $\rho_{11}^{0-}$ $\rho_{11}^{+-}$ $\rho_{10}^{+0}$ $\rho_{10}^{00}$ $\rho_{11}^{0-}$ $\rho_{11}^{+-}$ $\rho_{10}^{+0}$ $\rho_{10}^{00}$
$\rho_{11}^{+-}$ $\rho_{10}^{+0}$	L = 1	$A_2^{1+-}$ $\rho_{10}^{+0}$ $\rho_{10}^{00}$ $A_1^{1+-}$ $\rho_{10}^{+0}$ $\delta^{1+-}$ $\rho_{10}^{+0}$
$\rho_{11}^{+-}$	L = 0	$\rho_{11}^{+-}$ $\rho_{10}^{+0}$
$\underbrace{\rho_{11}^{+-} \quad \rho_{10}^{+0}}_{S=0}$		$\underbrace{\rho_{10}^{+0} \quad \rho_{10}^{00}}_{S=1}$

Fig. 4 Meson states in the new Gell-Mann-Zweig scheme. Solid lines: states allowed by the "old" quark model, including radial excitations, which are labelled n = 1. Dashed lines: new states, including daughters and parity doublets. L, S, and n refer to the quantum numbers of the "old" quark model. Every state corresponds to a J<sup>PC</sup>, SU(3) nonet.

parity states as well as for all the conventional S = 0 (unnatural parity) states. In addition they have a sequence of parity doublets for every one of the conventional S = 1 unnatural parity states (such as the A<sub>1</sub>) but these sequences terminate at J = 1. In terms of Toller's classification of trajectories, they assign the conventional S = 1 unnatural parity L = J states to M = 1 families, whilst all other states have M = 0.

The proliferation of new states in the Gell-Mann-Zweig model is especially noticed for the higher L-values, but even below 1.5 GeV they expect many new states (assuming that the daughter masses are approximately the same as their respective parent masses). The new "wanted" states below 1.5 GeV are<sup>19)</sup>:

- a) A J<sup>PC</sup> = 0<sup>+-</sup> nonet approximately degenerate with the ordinary vector mesons. Its non-strange members should decay to 4π or 5π, whilst the strange component might decay to Kπ [κ(730)??].
- b) A J<sup>PC</sup> = 0<sup>--</sup> nonet around the B-meson mass. The I = 1 state could decay to π+ω and might be "covered" by the B. The other states should be observed somewhere within the general confusion of the A<sub>1</sub>-A<sub>2</sub> and the Q-regions in the 3π and Kππ systems, respectively.
- c) A J<sup>PC</sup> = 1<sup>-+</sup> nonet, the parity doublet of the A<sub>1</sub> nonet.
- d) Another 1<sup>-+</sup> nonet, the "daughter" of the A<sub>2</sub> nonet. A<sub>2</sub><sup>L</sup>(1270) could belong to such a nonet if: i) A<sub>2</sub> splits; ii) both "halves" decay into πρ and πη; iii) only A<sub>2</sub><sup>H</sup> decays into K<sub>1</sub><sup>0</sup>K<sub>1</sub><sup>0</sup>. Very crude indications supporting such a possibility already exist<sup>20)</sup>, but we should definitely wait for more data.
- e) A 0<sup>++</sup> nonet around the 2<sup>++</sup> nonet mass.

The over-all experimental picture certainly does not support the Gell-Mann-Zweig proposal, but we cannot rule it out either, since all or most of their predicted low-lying states are not easily detectable.

Quite independently of this detailed model, the apparent absence of states belonging to daughter trajectories should concern those of us who believe that the "daughters" are roughly parallel to their "parents". A quantitative (bootstrap-type?) estimate of the couplings of daughter particles to various channels is badly needed.

### 2.3 Mesons and SU(3)

We now proceed to discuss the present status of SU(3) for meson resonances.

#### 2.3.1 $0^{-+}$ nonet

Using a quadratic mass formula, the  $\eta$ - $X^0$  octet-singlet mixing angle is  $\pm(10.4 \pm 0.2)^\circ$ . A linear mass formula would give  $\pm(24 \pm 1)^\circ$ . The unmixed  $\eta_8$  is a 2:1 mixture of strange and non-strange  $q\bar{q}$  pairs. A mixing angle  $\theta_0 = 10^\circ$  may raise the  $\lambda\bar{\lambda}$  content in the physical  $\eta$  particle to about 80% or reduce it to 50%, depending on the sign of  $\theta_0$ . Within the quark model the sign of the mixing angle can be determined from its effect on various decay processes. Assuming that the  $\pi^0$ ,  $\eta$ ,  $X^0 \rightarrow \gamma\gamma$  transition matrix elements (after extracting the appropriate phase space factors) satisfy the usual SU(3) relations, we find:

$$\Gamma(X^0 \rightarrow \gamma\gamma) = \frac{m_{X^0}^3}{\sin^2 \theta_0} \left[ \left( \frac{\Gamma(\pi^0 \rightarrow \gamma\gamma)}{3m_\pi^3} \right)^{\frac{1}{2}} \pm \left( \frac{\Gamma(\eta \rightarrow \gamma\gamma)}{m_\eta^3} \right)^{\frac{1}{2}} \cos \theta_0 \right]^2,$$

where the  $\pm$  signs in the equation correspond to the different possible signs for the mixing angle  $\theta_0$ . Assuming<sup>5)</sup>  $\Gamma(\pi^0 \rightarrow \gamma\gamma) = 7.5 \pm 1.5$  eV,  $\Gamma(\eta \rightarrow \gamma\gamma) = 0.88 \pm 0.22$  keV, and  $\theta_0 = \pm 10^\circ$ , we predict that  $\Gamma(X^0 \rightarrow \gamma\gamma) = 50 \pm 30$  keV or  $350 \pm 90$  keV, depending on the sign of  $\theta_0$ . A mixing angle of  $\theta_0 = 24^\circ$  would give  $10 \pm 6$  keV or  $90 \pm 20$  keV. A preliminary result for the  $X^0 \rightarrow \gamma\gamma$  branching ratio, presented at this Conference<sup>21)</sup>, is consistent with all these possibilities. The quark model, however, prefers the negative sign in the expression for  $\Gamma(X^0 \rightarrow \gamma\gamma)$ , predicting<sup>22)</sup>  $\Gamma(X^0 \rightarrow \gamma\gamma)$  around 50 keV. This corresponds to a 50%-50% mixture of strange and non-strange  $q\bar{q}$  pairs in the physical  $\eta$ . This conclusion is qualitatively supported by the approximately equal cross-sections for  $\eta$  and  $X^0$  production in high-energy  $\pi p$  reactions, which should be contrasted with the large  $\omega$  to  $\phi$  and  $f^0$  to  $f^*$  production ratios in  $\pi p$  collisions. The decay  $\delta \rightarrow \eta\pi$  leads, however, to an opposite conclusion concerning the sign of  $\theta_0$  (see Section 2.3.4).

#### 2.3.2 $1^{--}$ nonet

The  $\omega$ - $\phi$  mixing angle predicted by the quadratic mass formula is  $\theta_1 = 40^\circ$ . Apart from symmetry-breaking corrections, SU(3) invariance and the octet assignment of the photon lead to:

$$\Gamma(\rho^0 \rightarrow \ell^+\ell^-) : \Gamma(\omega \rightarrow \ell^+\ell^-) : \Gamma(\phi \rightarrow \ell^+\ell^-) = 3 : \sin^2 \theta_1 : \cos^2 \theta_1.$$

The recent measurements of these leptonic decay rates, reported at this Conference<sup>23)</sup>, give ratios 7:1:0.8 with 20%-40% errors. Symmetry-breaking effects may lead to corrections of factors of 2 to the above predictions. Most models for such corrections<sup>24)</sup> are consistent with the present preliminary data, and are discussed elsewhere in the proceedings<sup>25)</sup>.

The situation concerning the decays  $1^- \rightarrow 0^- + 0^-$  remains essentially unchanged. Assuming that the only phase-space correction is given by  $p_{C.M.}^{2\ell+1}$  and using<sup>5)</sup>  $\Gamma(K^* \rightarrow K\pi) = 49.7 \pm 1.1$  MeV, SU(3) predicts:  $\Gamma(\rho \rightarrow \pi\pi) \sim 130$  MeV,  $\Gamma(\phi \rightarrow K\bar{K}) \sim 5$  MeV. Most models for these decays would tend to correct the ratios between these decay rates by 20%-40%, following from the  $\rho$ ,  $K^*$ , and  $\phi$  mass differences. Within the ambiguities of such corrections the agreement is satisfactory. Notice, however, that most of the success here emerges from the phase-space factors!

A beautiful test of SU(3) for the coupling of the  $\rho$  trajectory to  $\pi\pi$  and  $K\bar{K}$  was suggested by Barger and Olsson<sup>25)</sup>. Using high-energy Regge parametrizations for  $\pi^-p$ ,  $K^-p$ , and  $K^+n$  charge-exchange scattering, they find excellent agreement between experiment and the SU(3) prediction for the ratio between the  $\rho\pi\pi$  and  $\rho K\bar{K}$  residue functions.

#### 2.3.3 $2^{++}$ nonet

Here there are new developments, related to the possible splitting<sup>27)</sup> of the  $A_2$ . The octet-singlet ( $f^0$ - $f^*$ ) mixing angle obtained from the quadratic mass formula is  $\theta_2 = 30^\circ$ . The following ratios among the various  $K^{**}(1420) \rightarrow 1^- + 0^-$  decay modes (assuming a  $p_{C.M.}^{2\ell+1}$  phase-space factor) are obtained:

$$\Gamma[K^{**} \rightarrow K^*(890) + \pi] : \Gamma[K^{**} \rightarrow \omega + K] : \Gamma[K^{**} \rightarrow \rho + K] \\ \sim 12:3:1.$$

The experimental ratios are approximately 10:3:1 with

10%-30% experimental errors, in addition to the usual SU(3) ambiguities. The agreement is excellent and cannot be obtained from phase space alone. Using the  $K^{**}(1420)$  decays as input, we then predict  $\Gamma(A_2 \rightarrow \rho + \pi) \sim 90$  MeV. If the  $A_2$  indeed splits and we have two different resonances,  $A_2^H$  and  $A_2^L$ , each having  $\Gamma \sim 30$  MeV, the discrepancy cannot be tolerated. No mixing or mass-dependent factors could be reasonably invoked here to account for the "missing" factor of 3, unless the "other"  $A_2$  is also a  $2^+$  state. In that case we may even have one relatively wide and one relatively narrow resonance interfering with each other<sup>2a)</sup>. Barring such a possibility, and accepting the splitting of the  $A_2$ , we are led to predict that  $K^{**}(1420)$  should also split. The predicted width for  $f^* \rightarrow K^*(890) + K$ , using the unsplit  $K^{**}(1420)$  as input, is around 15-20 MeV, whilst if  $K^{**}$  splits (or if we use  $A_2^H$  as input) we find 5 MeV. Experimentally,  $\Gamma(f^* \rightarrow K^*K) \leq 15$  MeV. Needless to say, the existence of a pair of  $A_2$ 's also predicts another pair of isoscalars in addition to  $f^0$  and  $f^*$ , but those need not be degenerate with  $f^0$  and  $f^*$ , and SU(3) does not "demand" the splitting of the known  $f$  particles.

Concerning the decays  $2^+ \rightarrow 0^- + 0^-$ , the comparison between SU(3) and experiment is displayed in Table 2. Assuming that  $K^{**}(1420)$  does not split, the agreement is significantly good. If, however,  $K^{**}(1420)$  splits,

either the entire  $K\pi$  mode will come from the  $2^+$  part of the split  $K^{**}$ , or  $f^0$  and  $f^*$  should also split in order to achieve even crude agreement with SU(3). Note that the details of this comparison do depend in a very sensitive way on the values of  $\theta_2$  and  $a$ , where  $a$  is the octet to singlet ratio of the decay matrix elements.

The  $A_2\eta\pi/A_2K\bar{K}$  ratio for the couplings of the  $A_2$  trajectory was compared with the high-energy data for  $\pi^-p \rightarrow \eta n$  and  $K^+n$  and  $K^-p$  charge exchange by Barger and Olsson<sup>26)</sup>. The agreement is extremely good.

Table 2 gives a very weak indication that the  $A_2 \rightarrow \eta\pi$  decay rate is slightly smaller than predicted by SU(3). This is by no means conclusive in view of the splitting problem and the small size of the discrepancy. If, however,  $A_2 \rightarrow \eta\pi$  really turns out to be significantly smaller than the SU(3) prediction, it could hint that the  $\eta-X^0$  mixing angle tends toward the almost pure  $\lambda\bar{\lambda}$  decomposition of  $\eta$ , contrary to our conclusion from the  $\eta \rightarrow \gamma\gamma$  rate. It would be interesting to watch future measurements of this decay rate (see Sections 2.3.1, 2.3.4, and 2.4).

#### 2.3.4 $0^{++}$ nonet

Assuming that the members of a  $0^{++}$  nonet are  $\sigma(750)$ ,  $K^*(1100)$ ,  $\delta(960)$ , and  $S^*(1070)$ , we find that no mixing angle can fit the mass formula, since the

TABLE 2

$2^+ \rightarrow 0^- + 0^-$  decays and SU(3).  $\theta_2 = 30^\circ$  as given by the quadratic mass formula.  $a = 2 \cot \theta_2$  is chosen on the basis of the small  $f^* \rightarrow \pi\pi$  decay rate. The only phase-space correction is a  $p_{c.m.}^{2L+1}$  factor.

Decay	C.G. Coefficient	Prediction	Experiment
$A_2 \rightarrow K\bar{K}$	12	6	< 5
$A_2 \rightarrow \eta\pi$	8	11	< 10
$f^0 \rightarrow \pi\pi$	$3(2 \sin \theta_2 + a \cos \theta_2)^2 = 48$	140	$145 \pm 25$
$f^0 \rightarrow \eta\eta$	$(2 \sin \theta_2 - a \cos \theta_2)^2 = 4$	1	$\sim 0$
$f^0 \rightarrow K\bar{K}$	$4(\sin \theta_2 - a \cos \theta_2)^2 = 25$	7	< 4
$K^{**} \rightarrow K\pi$	18	53	$45 \pm 4$
$K^{**} \rightarrow K\eta$	2	0.2	$2 \pm 1$
$f^* \rightarrow \pi\pi$	$3(2 \cos \theta_2 - a \sin \theta_2)^2 = 0$	0	< 10
$f^* \rightarrow \eta\eta$	$(2 \cos \theta_2 + a \sin \theta_2)^2 = 12$	15	< 30
$f^* \rightarrow K\bar{K}$	$4(\cos \theta_2 + a \sin \theta_2)^2 = 27$	53	$50 \pm 20$

$K^*$  state is the heaviest. The discrepancy can be removed only if the  $m(K^*) < 1070$ . The "usual" quark model angle  $\theta \sim 30^\circ\text{--}40^\circ$  would be a natural choice here, in view of the apparently weak  $S^* \rightarrow \pi\pi$  rate. This would predict  $m(K^*) \sim 960$ . Independently of the particular value of the mixing angle, if we assume  $S^* \not\rightarrow \pi\pi$  and  $\Gamma(S^* \rightarrow K\bar{K}) \sim 70$  MeV, we predict:

i)  $\Gamma(K^* \rightarrow K\pi) \geq 80$  MeV; ii)  $\Gamma(\sigma \rightarrow \pi\pi) \geq 500$  MeV; iii)  $\Gamma(\delta \rightarrow \eta\pi) \geq 40$  MeV. Whilst the first two predictions are not inconsistent with the information obtained from the  $\pi\pi$  and  $\pi K$  "phase-shift analysis"<sup>10,11)</sup>, the actual width of  $\delta(960)$  seems to be much smaller [the missing-mass spectrometer gives  $\Gamma(\delta) < 5$  MeV]. This can be understood within the quark model, if  $\eta$  is an almost pure  $\lambda\bar{\lambda}$  state; namely, if the sign of the  $\eta\text{-}X^0$  mixing angle is opposite to the one needed for explaining the strength of the  $\eta \rightarrow \gamma\gamma$  decay mode. The sign choice indicated by the  $\delta$  decay is further supported by chiral symmetry considerations (see Section 2.4). We do not know how to resolve these conflicting pieces of evidence.

### 2.3.5 $1^+$ nonet

Here we face the well-known problem of mixing between the two axial  $K^*$  mesons belonging to the two axial nonets<sup>13)</sup>. The large number of missing states and unknown mixing angles prevent us from drawing any significant conclusions. If the octet-singlet mixing angles in both axial nonets are the usual quark model angles ( $\arccos \sqrt{2/3}$ ) and the decays  $B \rightarrow \phi\pi$ ,  $b' \rightarrow \rho\pi$  (where  $b'$  is the  $\lambda\bar{\lambda}$  isoscalar of the B nonet) are forbidden, we can compute the transition rates of all  $1^+ \rightarrow 1^- + 0^-$  processes in terms of  $\Gamma(A_1 \rightarrow \rho\pi)$ ,  $\Gamma(B \rightarrow \omega\pi)$ , and the  $K_1^*-K_2^*$  mixing angle  $\lambda$ . Our results are given in Table 3. For  $\lambda = 0$  we obtain reasonable results for the  $K^*$  width, but a small variation in  $\lambda$  can change the picture. Note that if  $b' \not\rightarrow \rho\pi$ , its decay to  $K^*K$  will be relatively weak and the only allowed mode would probably be  $K\bar{K}\pi$ . The  $1^{+-}$  isoscalar that does decay to  $\pi\rho$  should have a width of the order of 100 MeV. The phase-space corrections in this case are even more ambiguous than usual in view of the al-

TABLE 3

Decay rates for the two axial nonets (assuming a  $K_1^* - K_2^*$  mixing angle  $\lambda$ ).  $a$  and  $b$  are the "non-strange"  $q\bar{q}$  isoscalar states, and  $a'$ ,  $b'$  are the "strange" ( $\lambda\bar{\lambda}$ ) isoscalar  $q\bar{q}$  states in the  $A_1$  and B nonets, respectively. The decays  $B \rightarrow \phi\pi$  and  $b' \rightarrow \rho\pi$  are assumed to be forbidden.

Decay	C.G. Coefficient	Remarks
$A_1 \rightarrow \rho\pi$	$2 f^2$	Probably not allowed [ $m(m, a') < 1.39$ GeV]
$a \rightarrow K^*K$	$\frac{1}{2} f^2$	
$a' \rightarrow K^*K$	$f^2$	
$B \rightarrow \omega\pi$	$2 g^2$	$120 \pm 20$ (input)
$b \rightarrow \rho\pi$	$2 g^2$	$120 \pm 20$ [if $m(b) = m(B)$ ]
$b \rightarrow K^*K$	$\frac{1}{2} g^2$	$< 10$ MeV if $m(b) < 1.55$
$b' \rightarrow K^*K$	$g^2$	$< 20$ MeV if $m(b') < 1.55$
$K_1^* \rightarrow \rho K$	$(f \cos \lambda - g \sin \lambda)^2$	Depend very sensitively on sign and magnitude of $\lambda$ . $K_{1,2}^* \rightarrow \rho K / \omega K = 2$ independent of $\lambda$ .
$K_1^* \rightarrow \omega K$	$\frac{1}{2} (f \cos \lambda - g \sin \lambda)^2$	
$K_1^* \rightarrow K^*\pi$	$(f \cos \lambda + g \sin \lambda)^2$	
$K_2^* \rightarrow \rho K$	$(f \sin \lambda + g \cos \lambda)^2$	
$K_2^* \rightarrow \omega K$	$\frac{1}{2} (f \sin \lambda + g \cos \lambda)^2$	
$K_2^* \rightarrow K^*\pi$	$(f \sin \lambda - g \cos \lambda)^2$	

lowed S-wave and D-wave decays, with an a priori unknown relative strength.

#### 2.4 Chiral $SU(2) \times SU(2)$ and the meson classification

Chiral  $SU(2) \times SU(2)$  has emerged in the last few years as a useful dynamical symmetry<sup>29)</sup>, leading directly to relations among weak and electromagnetic matrix elements, and via PCAC and the vector meson dominance to predictions for strong interaction processes. The classification of particle states in the infinite momentum frame into representations of chiral  $SU(2) \times SU(2)$  leads to many interesting relations amongst masses and coupling constants<sup>30)</sup>. Most of these relations are particularly relevant to the spectrum of low-lying mesons. We now list some of the consequences of the  $SU(2) \times SU(2)$  classification, which are of immediate experimental interest.

a) Current algebra sum rules for  $\pi\delta(960)$  elastic scattering indicate that an interesting relation exists between the  $\delta$  and  $X^0$  mesons. A single assumption<sup>30)</sup> on the classification of  $\delta$  and  $X^0$  into a  $(1/2, 1/2)$  multiplet of  $SU(2) \times SU(2)$  predicts  $m(\delta) = m(X^0)$  and  $\delta \neq \eta\pi$ . Both predictions are approximately correct since the  $\delta$  width is extremely small. This relation between the tiny  $\delta$ - $X^0$  mass difference and the "almost forbidden"  $\delta \rightarrow \eta\pi$  decay is extremely interesting. If  $X^0$  mostly belongs to a  $(1/2, 1/2)$  chiral representation, the  $\eta$  is probably in a  $(0, 0)$ , and in terms of the quark model we are again led to identifying  $\eta$  as a  $\lambda\bar{\lambda}$  state. The existence of the transition  $D \rightarrow \delta\pi$ <sup>9)</sup> probably means that there is some component of the D meson in the same  $(1/2, 1/2)$  representation, mixed with  $X^0$ . Notice that particles of different spins (but the same helicity) can mix in  $SU(2) \times SU(2)$ .

b) The  $\sigma$  meson is predicted<sup>30)</sup> to be approximately degenerate with  $\rho$ , in agreement with the recent  $\pi\pi$  analysis. They should satisfy

$$m_\sigma^2 = m_\rho^2 = \frac{1}{2}(m_\pi^2 + m_{\Lambda_1}^2).$$

c) The decay  $A_1 \rightarrow \rho + \pi$  should proceed mainly via the longitudinal mode<sup>30)</sup>; namely, the  $1 \leftrightarrow 1$  helicity amplitude should vanish. This means that there is more D-wave than S-wave contribution to the decay. A recent SLAC experiment<sup>31)</sup>, submitted to the Confer-

ence verifies this, but more data are definitely needed.

d)  $A_1$  should have a strong  $\pi + \sigma$  decay mode with a width of the order of 50 MeV. It is almost impossible to distinguish between this mode and the  $\pi + \rho$  mode, except for two cases: i)  $A_1 \not\rightarrow \rho^0 + \pi^0$  but  $A_1 \rightarrow \sigma + \pi^0$ ; ii) the apparent  $(A_1^+ \rightarrow \rho^+ \pi^0)/(A_1^+ \rightarrow \rho^0 \pi^+)$  branching ratio should be smaller than 1, because some of the " $\rho^0$ " events should really be  $\sigma$  events. Both features are difficult to observe experimentally.

#### 2.5 Other theoretical models for mesons

A variety of theoretical ideas have been applied to the calculation of meson decay rates and masses. Some of their most interesting aspects (from the point of view of experimental tests) are the following:

- a) Hard-pion techniques lead to predictions for  $A_1 \rightarrow \pi\rho$  and  $A_1 \rightarrow \pi\sigma$ .  $A_1 \rightarrow \pi\rho$  is supposed to be dominated by S-wave<sup>32)</sup>, in contradiction with the sum rule prediction of the previous section and with the preliminary experimental indications<sup>31)</sup>.  $A_1 \rightarrow \pi\sigma$  is again predicted<sup>33)</sup> to have a substantial decay rate.
- b) Quark-model calculations of pion emission by quarks, neglecting recoil corrections, predict<sup>34)</sup> that  $A_1 \rightarrow \rho\pi$  and  $B \rightarrow \omega\pi$  are not dominated by S-wave, that the first is purely transverse and the second purely longitudinal. Both predictions seem to contradict the data.  $B \rightarrow \omega\pi$  is consistent with a purely transverse decay, according to a recent Illinois experiment<sup>35)</sup>.
- c) Superconvergence relations and finite energy sum rule bootstrap calculations lead to relations amongst meson masses such as<sup>36)</sup>  $m(B) = m(A_2) - m(\pi)$ ,  $m^2(A_2) = 3[m^2(\rho) - m^2(\pi)]$ , in good agreement with experiment.

### 3. BARYON SPECTROSCOPY

#### 3.1 The status of baryon $SU(3)$ multiplets

The most significant development reported to this Conference with respect to the baryon spectrum is the discovery of several new  $Y^*$  candidates<sup>2)</sup> which fall very neatly into the various  $SU(3)$  octets and decuplets "started" by the "old"  $N^*$  resonances<sup>3)</sup>. We first discuss the existence of the various states in these multiplets and then proceed in Section 3.2 to analyse, in more detail, decay rates, possible representation mixing, etc.

The known  $\ell \leq 3$  N states ( $I = 1/2$ ) are displayed in Fig. 5. Those include all the  $I = 1/2$   $\pi N$  states claimed by the CERN group last fall<sup>37)</sup>, as well as the possible  $D_{13}(1730)$  state that was hinted by their analysis but was not claimed to be a resonance. In agreement with Donnachie<sup>3)</sup>, we classify  $D_{13}(1730)$ ,  $F_{17}(1980)$ , and  $D_{13}(2060)$  as being shakier than the other states. Assuming that the only "allowed" SU(3) multiplets are singlets, octets, and decuplets, we are led to believe that every excited nucleon state "starts" an octet. We therefore expect three other states ( $\Lambda$ ,  $\Sigma$ , and  $\Xi$ ) for every one of the 12 N states of Fig. 5 (if they really exist).

The spectrum of excited  $\Delta$  states ( $I = 3/2$ ) is shown in Fig. 6 where, again, all the "products" of the CERN 1967 analysis<sup>37)</sup> are included, with  $D_{35}(1950)$  and  $P_{33}(1690)$  marked<sup>3)</sup> as "dubious". Following the argument of the previous paragraph, we expect three additional states ( $\Sigma$ ,  $\Xi$ , and  $\Omega$ ) for every  $\Delta$ , assuming that all  $\Delta$ 's belong to decuplets.

The known excited  $\Lambda$  states<sup>2)</sup> are listed in Fig. 7. They include the new possible  $P_{01}(1750)$  state found by the CHS Collaboration<sup>38)</sup> and the "older"  $F_{07}(1870)$  possible resonance. Since the  $I = 0$   $Y^*$ 's may a priori belong to SU(3) singlets or octets, we listed for every one of them a tentative assignment.  $\Lambda(1405)$  and  $\Lambda(1520)$  are known to be mostly in SU(3) singlets, although some octet-singlet mixing may be needed (see Section 3.2). We believe that all the other  $\Lambda$  states

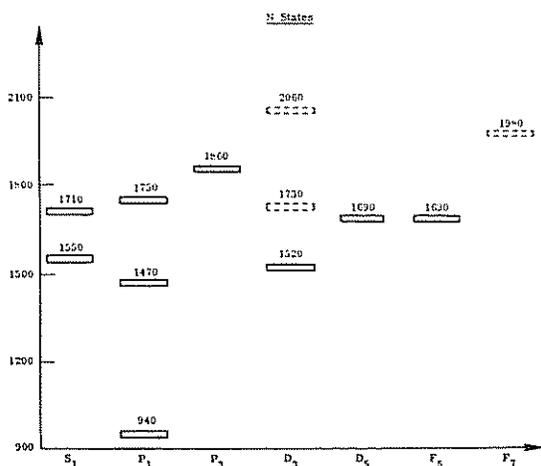


Fig. 5 The excited N states ( $I = 1/2$ ). Solid boxes represent relatively well-established states. Dashed boxes represent states which are more uncertain.

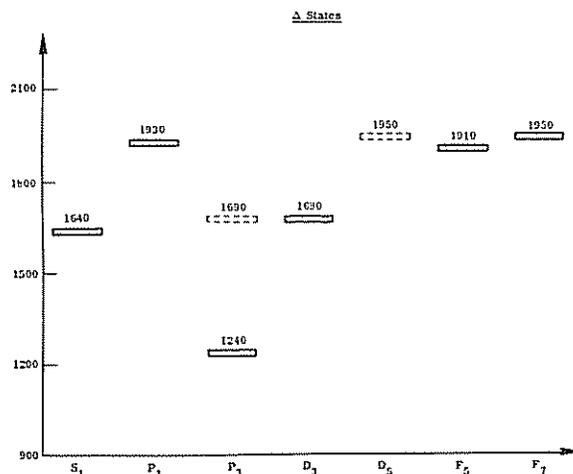


Fig. 6 The excited  $\Delta$  states ( $I = 3/2$ ). Notation as in Fig. 5.

of Fig. 7, with the possible exception of the "peculiar"  $F_{07}(1870)$ , are in octets. Our discussion (below) of the existing octets seems to support these assignments.

The  $\Sigma$  states<sup>2)</sup> are given in Fig. 8, and they include the following new assignments:

- a) A possible  $S_{11}(1660)$  state was found by the CHS group<sup>38)</sup> in  $K^-p \rightarrow \Lambda\pi$ . This may be the same state as the  $Y_1^*(1680)$  found last year by a Northwestern-Argonne group<sup>39)</sup>. We label it  $S_{11}(1670)$  in the figure.
- b) The old  $\Sigma\eta$  enhancement at 1770 is still in doubt<sup>2)</sup>. This is our  $S_{11}(1770)$  state.
- c) A new  $P_{11}(1610)$  state is hinted by the CHS  $K^-p \rightarrow \Lambda\pi$  data<sup>38)</sup>. This may be the same state as the  $Y_1^*(1616)$  reported at this Conference by a BNL group<sup>40)</sup>.

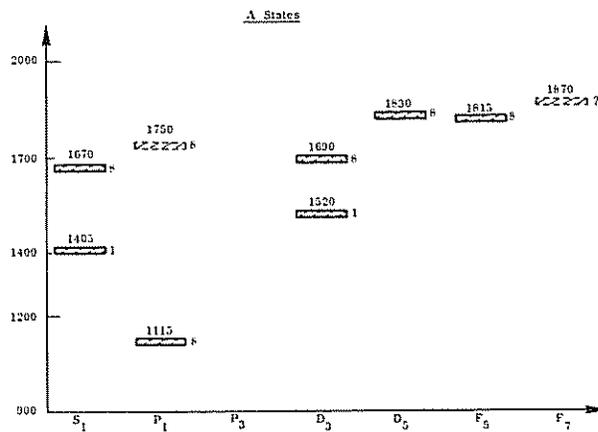


Fig. 7 The excited  $\Lambda$  states. Notation as in Fig. 5. Singlet or octet assignments are indicated for every state.

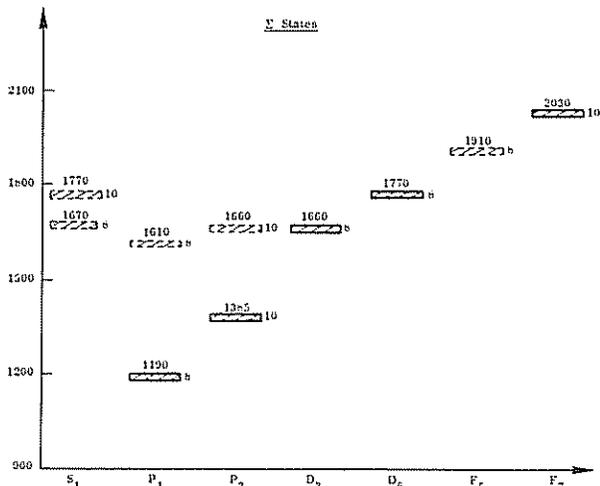


Fig. 8 The excited  $\Sigma$  states. Notation as in Fig. 5. Octet or decuplet assignments are indicated for every state.

d) The CHS group also reports a possible  $P_{13}(1660)$  state which they find in their  $K^+p \rightarrow \Lambda\pi$  analysis<sup>3B)</sup>.

e) The  $F_{15}(1910)$  state is not confirmed<sup>2)</sup> and is listed here "with reservations".

All  $\Sigma$  states should probably belong to octets or decuplets. In addition to the old assignments of the established states we suggest that  $S_{11}(1670)$  and  $S_{11}(1770)$  are octet-decuplet mixtures, with  $S_{11}(1670)$  being mostly in the octet and  $S_{11}(1770)$  mostly in the decuplet.  $P_{11}(1610)$  is probably in the octet of the  $N(1470)$  resonance and  $P_{13}(1660)$  should perhaps be in the decuplet of the "second"  $3,3$  resonance  $\Delta(1690, 3/2^+)$ .

With these assignments we can now proceed to study the present status of the various octets and decuplets. For this purpose we ignore the possible mixing effects and tentatively assume that the  $S = -1$  members of every multiplet should be found approximately 100-200 MeV above the mass of the non-strange baryon in the multiplet. This assumption does not follow directly from any  $SU(3)$  considerations. It may be "derived" from simple quark-model arguments (the  $\lambda$  quark is heavier than the non-strange quarks by 100-200 MeV?) or can be based on our experience with those  $SU(3)$  multiplets which are already established. Mixing effects, whilst drastically changing the decay rate predictions, usually do not affect the baryon masses by more than 50 MeV, well within the range of our rough

approximation. In order to examine the  $SU(3)$  octets we have plotted all  $N$ ,  $\Lambda$ , and  $\Sigma$  states which we had assigned to octets in Figs. 5, 7, and 8 on three different mass scales, shifted with respect to each other. This is shown in Fig. 9 where the  $N$  spectrum,  $\Lambda$  spectrum (shifted by about 200 MeV), and  $\Sigma$  spectrum (shifted by about 220 MeV) are plotted together. The grouping of  $N$ ,  $\Lambda$ , and  $\Sigma$  states to  $SU(3)$  octets is striking. Below 1850 MeV for  $\Sigma$  and  $\Lambda$  masses, or 1700 MeV for  $N$

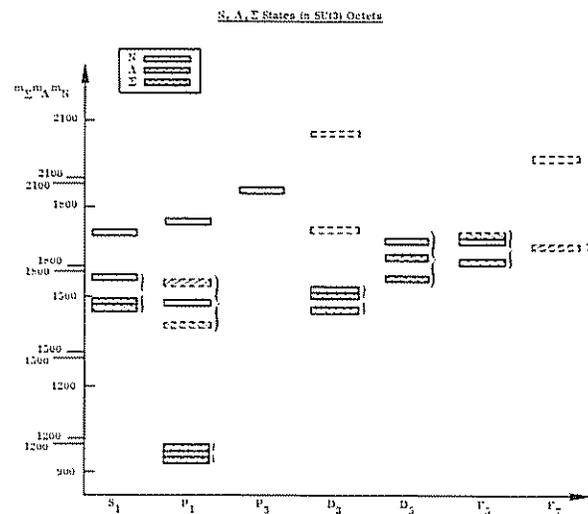


Fig. 9  $N$ ,  $\Lambda$ , and  $\Sigma$  excited states assigned to  $SU(3)$  octets. The three mass scales are shifted with respect to each other. The "ideal world" in which the  $N-\Lambda-\Sigma$  mass splitting in all octets is identical would correspond to overlapping "boxes" for the corresponding states in a given octet. The solid and dashed boxes are defined in Fig. 5.

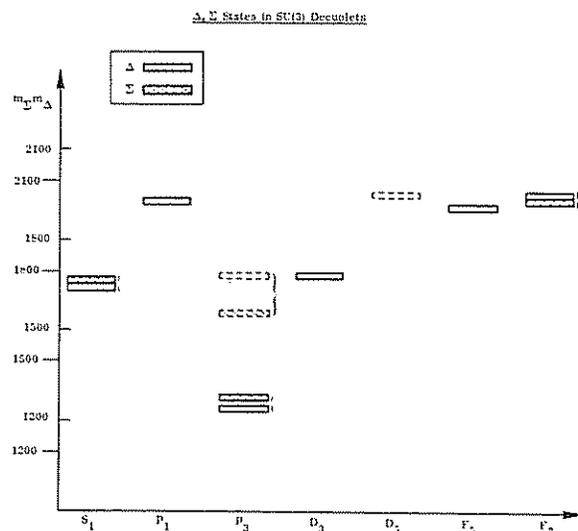


Fig. 10  $\Delta$  and  $\Sigma$  excited states assigned to  $SU(3)$  decuplets. Notation as in Fig. 9.

TABLE 4

Possible SU(3) octets. Unconfirmed states or questionable SU(3) assignments are question-marked.

$J^P$	N	$\Lambda$	$\Sigma$	$\Xi$
$\frac{1}{2}^+$	940	1115	1190	1320
$\frac{1}{2}^+$	1470	1750?	1610?	-
$\frac{3}{2}^-$	1520	1690	1660	1820?
$\frac{1}{2}^-$	1550	1670	1670?	-
$\frac{5}{2}^-$	1690	1830	1770	1930?
$\frac{5}{2}^+$	1690	1815	1910?	2030?

masses, all the octets are completed as far as their N,  $\Lambda$ , and  $\Sigma$  content is concerned. We will return to the missing excited  $\Xi$  states later. The newly discovered<sup>2)</sup>  $\Lambda$  and  $\Sigma$  states turn out to be exactly the states needed for completing all the expected octets! This is a remarkable success for SU(3).

Assuming that this successful classification pattern continues into the higher mass region we may predict future  $\Lambda$  and  $\Sigma$  states between 1850 and 2200 MeV. We expect the following states:

- a)  $\Lambda\left(1900, \frac{1}{2}^-\right)$ ;  $\Lambda\left(1950, \frac{1}{2}^+\right)$ ;  $\Lambda\left(2050, \frac{3}{2}^+\right)$ ;  $\Lambda\left(1900, \frac{3}{2}^-\right)$ ;  $\Lambda\left(2250, \frac{3}{2}^-\right)$  in octets.
- b) Additional possible SU(3) singlets.
- c)  $\Sigma\left(1900, \frac{1}{2}^-\right)$ ;  $\Sigma\left(1950, \frac{1}{2}^+\right)$ ;  $\Sigma\left(2050, \frac{3}{2}^+\right)$ ;  $\Sigma\left(1900, \frac{3}{2}^-\right)$ ;  $\Sigma\left(2250, \frac{3}{2}^-\right)$  in octets.
- d)  $\Sigma\left(2100, \frac{1}{2}^+\right)$ ;  $\Sigma\left(1850, \frac{3}{2}^-\right)$ ;  $\Sigma\left(2100, \frac{5}{2}^- \right)$ ;  $\Sigma\left(2050, \frac{5}{2}^+\right)$  in decuplets.

The masses of these predicted states should be within a range of 100 MeV or so around the above predictions.

The situation concerning the excited  $\Xi$  states is very unpleasant. Since every octet and every decuplet requires an  $I = 1/2$   $\Xi$  state, we could get a vague impression of the expected  $\Xi$  spectrum by combining the observed N and  $\Delta$  states on the same plot. The predicted  $\Xi$  states should probably lie 250-400 MeV above the corresponding non-strange resonances. Figure 11

TABLE 5

Possible SU(3) decuplets. Notation as in Table 4.

$J^P$	$\Delta$	$\Sigma$	$\Xi$	$\Omega$
$\frac{3}{2}^+$	1240	1385	1530	1675
$\frac{1}{2}^-$	1640	1770?	-	-
$\frac{3}{2}^+$	1690?	1660?	-	-
$\frac{7}{2}^+$	1950	2030	-	-

A similar analysis for the SU(3) decuplets indicates (Fig. 10) that the lowest missing  $\Sigma$  state should, again, be around 1850 MeV. Here we have a clear deviation from our naive  $\Sigma$ - $\Delta$  "spacing rule". The  $\Sigma(1660)$  state ( $J^P = 3/2^+$ ) is actually lower than its probable companion  $\Delta(1690)$ . Neither of them are established, however, and their masses might change in the final analysis.

The complete list of existing octets and decuplets is given in Tables 4 and 5.

indicates, for example, that somewhere between 1950 and 2050 MeV there should exist eight different  $\Xi$  states. Since a direct phase-shift analysis is not feasible here, it will be extremely difficult to disentangle the various  $\Xi$  baryons in this region. We must conclude here that states like  $\Xi(1820)$ ,  $\Xi(1930)$ , and the new  $\Xi(2030)$  reported to this Conference by a BNL group<sup>41)</sup>, are really complicated superpositions of many  $\Xi$  resonances, and their measured branching ratios represent weighted averages (with possible interference

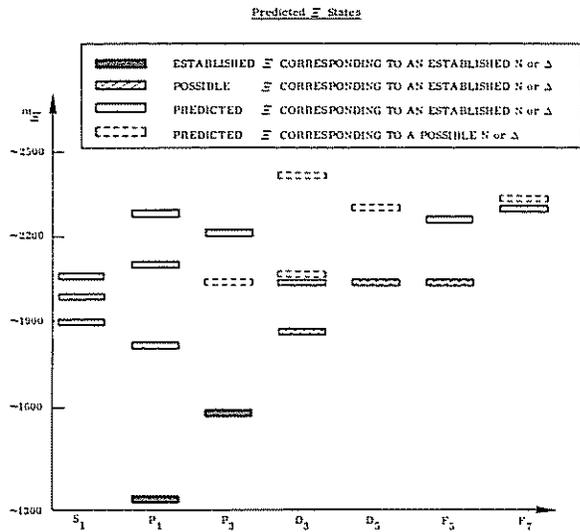


Fig. 11 The predicted  $\Sigma$  spectrum. The approximate mass scale is based on adding 300–350 MeV to the mass of the corresponding N or  $\Delta$  state.

effects in some cases) of the true decay rates of the various single states.

### 3.2 Branching ratios and representation mixing for baryon resonances in SU(3)

#### 3.2.1 $J^P = 1/2^-$

The existence of N(1550), N(1710), and  $\Delta(1640)$  leads us to assume that we have two octets and one decuplet with  $J^P = 1/2^-$ . The strongest indication for mixing is given by the prediction that, independently of the D/F ratios, any unmixed  $\Sigma$  state should have equal transition matrix elements to  $\Sigma\eta$  and  $\Lambda\pi$ , whilst any unmixed  $\Lambda$  state should have a 3:1 ratio between its  $\Sigma\pi$  and  $\Lambda\eta$  matrix elements. Phase-space considerations strongly favour the  $\Lambda\pi$  over the  $\Sigma\eta$  mode, and the  $\Sigma\pi$  over the  $\Lambda\eta$  mode. The unmixed  $\Sigma$  and  $\Lambda$  states should therefore always strongly prefer their  $\pi$ -emission decays over  $\eta$ -emission. These predictions are totally inconsistent with the decay branching ratios of  $\Sigma(1770)$  and  $\Lambda(1670)$ , both of which have an  $\eta$ -decay mode which is comparable in strength to the  $\pi$ -emission mode<sup>2)</sup>. This difficulty can be avoided either by inventing a symmetry-breaking correction which would balance the preference of the phase-space factor for the  $\Sigma\pi$  and  $\Lambda\pi$  decays<sup>4,2)</sup>, or by introducing strong representation mixing effects. The latter possibility is much more natural, especially since we do not see any reason why such nearby multiplets having the same  $J^P$  assignments

should not mix. Note that if we believe the simple mass-prescription of our previous discussion, we have to conclude that  $\Sigma(1670)$  is mostly in the same octet with the  $N\eta$  and  $\Lambda\eta$  "threshold resonances" N(1550) and  $\Lambda(1670)$ , whilst the  $\Sigma\eta$  "threshold state"  $\Sigma(1770)$  is at least partly in the  $\Delta(1640)$  decuplet. Note also that if  $\Sigma(1670)$  is purely in the same octet with  $\Lambda(1670)$ , we expect  $\Gamma[\Sigma(1670) \rightarrow \Lambda\pi] \sim 7\Gamma[\Lambda(1670) \rightarrow \Lambda\eta]$  after phase-space correction and independent of the D/F ratio. The experimental numbers are 110 and 18 MeV, respectively, in agreement with this prediction. Our mixing scheme for the  $1/2^-$  states should therefore avoid spoiling this ratio.

We do not expect to benefit from trying an overall fit for all the decay modes of these three multiplets, before at least all the missing excited  $\Lambda$  and  $\Sigma$  states are found (presumably around 1900 MeV).

$\Lambda(1405)$  is presumably mostly in an SU(3) singlet. Its coupling to  $\bar{K}N$  is much larger<sup>4,3)</sup> than to  $\Sigma\pi$ , and some mixing with other  $1/2^- \Lambda$  states may be necessary.

#### 3.2.2 $J^P = 1/2^+$

In addition to the established nucleon octet we expect two more octets [corresponding to N(1470) and N(1750)] and a decuplet for  $\Delta(1930)$ . Of these, only the first octet can be discussed in any detail. The recently discovered candidates<sup>3,8)</sup> for the second  $1/2^+$  octet are  $\Sigma(1610)$  and  $\Lambda(1745)$ . For the purpose of a preliminary, tentative, SU(3) analysis of this octet we assume: i) there are no mixing effects; ii) the D/F ratio for the decay rates of this octet is approximately the same as that of the basic baryon octet, i.e.  $D/F \sim 1.5-2$ . Using as input  $\Gamma[N(1470) \rightarrow N\pi] \sim 130 \pm 30$  MeV and introducing a  $(p_{c.m.})^{2L+1}$  factor as the only phase-space correction we predict

$$\begin{aligned} \Gamma[\Sigma(1610) \rightarrow \Lambda\pi] &\sim 15-30 \text{ MeV}; & \Gamma[\Sigma(1610) \rightarrow \Sigma\pi] &\sim 10-20 \text{ MeV}; \\ \Gamma[\Sigma(1610) \rightarrow N\bar{K}] &\sim 5 \text{ MeV}; & \Gamma[\Lambda(1745) \rightarrow \Sigma\pi] &\sim 70-130 \text{ MeV}; \\ \Gamma[\Sigma(1745) \rightarrow \Lambda\eta] &\sim 5 \text{ MeV}; & \Gamma[\Lambda(1745) \rightarrow N\bar{K}] &\sim 80-140 \text{ MeV}; \end{aligned}$$

As far as we can tell, all these numbers are not inconsistent with the experimental situation<sup>2)</sup>. The only "suspicious" prediction here is the large  $\Sigma\pi$  decay rate of  $\Lambda(1745)$ . The presence or absence of this state in the  $K^-p \rightarrow \Sigma\pi$  phase shifts will be a crucial

test of our simple picture. So far  $\Lambda(1745)$  has been seen<sup>3e)</sup> only in  $K^-p \rightarrow \bar{K}N$ .

A few years ago it was proposed<sup>4c)</sup> that  $N(1470)$  might belong to an antidecuplet of  $SU(3)$ . If this were the case, the decay  $N(1470) \rightarrow \Delta\pi$  would be forbidden in the exact  $SU(3)$  limit, and  $\gamma p \rightarrow N^+(1470)$  would be forbidden, whilst  $\gamma n \rightarrow N^0(1470)$  would be allowed<sup>45)</sup>. The apparent absence of the transition  $\gamma n \rightarrow N^0(1470)$  reported at this Conference<sup>46)</sup>, together with the observation of the  $\Delta\pi$  decay mode<sup>3)</sup>, suggest that the octet assignment is favoured.

### 3.2.3 $J^P = 3/2^+$

There is no news concerning the well-established  $3/2^+$  decuplet. A second  $3/2^+$  decuplet may be emerging, including  $\Delta(1690)$  and  $\Sigma(1660)$ . The existence of  $N(1860)$  predicts a  $3/2^+$  octet in the 1850–2250 MeV region. Very little is known about any of these states.

### 3.2.4 $J^P = 3/2^-$

$\Delta(1690)$  and the possible  $N(1730)$  and  $N(2060)$  are still waiting for their companions in a  $3/2^-$  decuplet and two  $3/2^-$  octets to be discovered. In addition, we now have an established  $3/2^-$  octet and a singlet. The octet-singlet mixing angle between  $\Lambda(1520)$  and  $\Lambda(1690)$  is probably around  $20^\circ$ , if the relevant  $\Xi$  state is indeed at 1820. Note that such a mixing angle may drastically change the various branching ratio predictions, whilst the mass values are changed only by 20 MeV. The  $\Lambda(1520)$  decay rates are actually inconsistent with a pure  $SU(3)$  singlet assignment<sup>47)</sup>, a fact which supports the necessity of octet-singlet mixing. The most interesting new development concerning the  $3/2^-$  octet is the observation of the  $\Xi(1820) \rightarrow \Sigma\bar{K}$  decay mode, reported by the BNL group<sup>41)</sup>. The apparent absence of this mode used to be the only definite failure of  $SU(3)$  in this octet, and this difficulty is now removed. Remember, however, that three  $\Xi$  states are expected around 1800 MeV, and we cannot be sure what we mean when we talk about " $\Xi(1820)$ ".

### 3.2.5 $J^P = 5/2^-$

No new major development has been reported, and the general situation is satisfactory. No prominent failures of the  $SU(3)$  predictions are encountered for

the  $5/2^-$  octet<sup>2,48)</sup>. If  $\Delta(1950)$  exists, we expect a  $5/2^-$  decuplet.

### 3.2.6 $J^P = 5/2^+$

The most troublesome difficulty<sup>48)</sup> in the  $5/2^+$  octet is the apparent absence of the  $\Sigma(1910) \rightarrow \Sigma\pi$  mode, which is predicted to have a partial width of about 50 MeV. The new  $\Xi(2030)$  which is probably a superposition of many states, may be mostly composed of the missing  $\Xi$  of the  $5/2^+$  octet<sup>41)</sup>, in the same way that the ancient "third nucleon resonance" at 1680 MeV represents many different  $N^*$ 's but is dominated by the  $5/2^+$  state. A  $5/2^+$  decuplet is expected by the  $\Delta(1910)$ .

### 3.2.7 $J^P = 7/2^+$

The  $7/2^+$  decuplet [ $\Delta(1950)$ ,  $\Sigma(2030)$ ] indicates no major difficulties in the decay rate analysis<sup>48)</sup>. If  $\Lambda(1860)$ ,  $7/2^+$  exists it should probably be an  $SU(3)$  singlet, unless we are prepared to associate it with the heavier possible  $N(1980)$ .

## 3.3 Baryon resonances and the quark model

The description of baryon states in terms of the quark model proceeds along the following lines:

a) The basic assumption states that at least the "low-lying" baryons are constructed from three quarks. This immediately predicts the existence of  $SU(3)$  singlets, octets, and decuplets, and the absence of any other  $SU(3)$  multiplet. The open question at this stage is: Do we have "exotic baryons" ( $I = 2$ , or  $S = +1$ , or  $I = 5/2$ , etc.), and if we do, how far above the "ground state" of the  $3q$  system will we find the first  $q\bar{q}$  excitation.

Experimentally, all the states listed in the previous sections can be accommodated in singlets, octets, and decuplets. The existence of "exotic baryons" other than the  $S = +1$ ,  $I = 0, 1$  states is not claimed by any experiment at this time. The question of whether the  $Z_1$  and  $Z_0$  "bumps" in the  $K^+p$  and  $K^+n$  total cross-sections are really resonances remains open<sup>2)</sup>, and will have to await a detailed  $KN$  phase-shift analysis. Such an analysis, in turn, necessitates much better polarization information on this process. We will return to the theoretical aspects of the possible existence of  $Z$  states in Sections 4.4(e), and 4.5(a), (b).

b) The next step is to assume a well-defined over-all symmetry for the entire  $3q$  wave function. There are models in which the three quarks are in a totally anti-symmetric state, whilst others prefer the symmetric possibility<sup>4,9)</sup>. Regardless of the specific symmetry property chosen, the states will have definite over-all symmetry with respect to the quark-spin and the SU(3) indices, and will therefore "fall" into SU(6) multiplets. SU(6) here is not necessarily an invariance group of any kind. It may simply be a device for counting all states having a specific over-all symmetry for their spin and SU(3) indices. The allowed SU(6) representations are the 56, 70, and 20, corresponding, respectively, to total symmetry, mixed symmetry, and total antisymmetry among the spin +SU(3) properties of the three quarks. This leads, in all versions of the quark model, to the grouping of states into SU(6) multiplets with a definite "orbital angular momentum"  $L$  and definite parity. At this stage, still without specifying the statistics that the quarks should obey, we can look at the baryon spectrum and try to identify  $[SU(6), L^P]$  supermultiplets. Figure 12 illustrates the established as well as the more speculative SU(3) multiplets. The positive parity baryons form two  $[56, 0^+]$  supermultiplets, a  $[56, 2^+]$  with one decuplet ( $J^P = 3/2^+$ ) missing and, presumably, a third  $[56, 0^+]$  in which only one excited nucleon state  $P_{11}(1750)$  exists so far. The low-lying negative

parity baryons fall very neatly into a  $[70, 1^-]$  multiplet with no missing SU(3) multiplets and no redundant states. This is a remarkable success for this extremely naive quark picture. The possibility of grouping such a large number of SU(3) multiplets with the correct spins and parities to 4 or 5  $[SU(6), L^P]$  multiplets is definitely a non-trivial matter, and we should probably take this success very seriously. Whether the physical picture of quarks is correct, or whether it just reflects some deeper dynamical structure, we do not know.

c) The third stage in developing this kind of quark description is to specify the type of statistics that the quarks obey, and to try to understand the "force" or "potential" which is responsible for the observed baryon states. In the last few years the symmetric quark model, first proposed by Greenberg<sup>50)</sup>, emerged as the most hopeful version of the quark scheme. The antisymmetric (ordinary fermions) model is not completely ruled out<sup>4,9)</sup>, but it is much more complicated and has no other virtues except for the familiarity of ordinary Fermi statistics.

The symmetric quark model assumes that the three-quarks possess a totally symmetric wave function. There are two almost equivalent ways of achieving this: i) assume that the quarks are parafermions of order three<sup>50)</sup>; ii) assume that we have three integral-charge triplets [the Han-Nambu model<sup>51)</sup>]. The possibility of paraquarks, which was proposed by Greenberg in 1964, has the advantage of being so unfamiliar that we may even try to blame the apparent non-existence of free quarks on some mysterious understood property of parafermions. (Can free parafermions exist?)

Various details of these different models are discussed elsewhere in these proceedings<sup>4,9)</sup>. Here we would like to discuss very briefly only one aspect of the symmetric model -- the sequence of  $[SU(6), L^P]$  multiplets predicted by an harmonic-oscillator shell-model description of an over-all symmetric  $3q$  system. Table 6 indicates the sequence of single and double quantum excitations predicted by such a model (all "spurious multiplets" corresponding to motion of the centre of mass are removed). The first four multiplets listed in the table are precisely those which

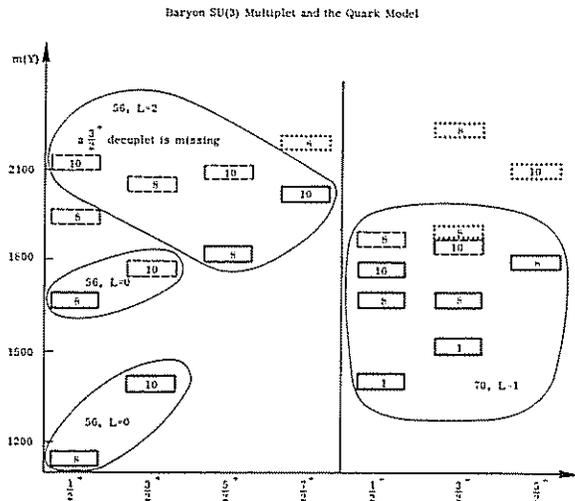


Fig. 12 The quark model  $[SU(6), L^P]$  supermultiplets. The mass scale corresponds to the approximate mass of the  $S = -1$  members of the various SU(3) singlets, octets, and decuplets. The solid, dashed, and dotted boxes correspond to probable, possible, and doubtful SU(3) multiplets, respectively.

TABLE 6

Single and double quantum excitations in the harmonic oscillator shell model for the symmetric  $3q$  system.

Shell Model State	SU(6)	L	Parity
$(1s)^3$	56	0	+
$(1s)^2 (1p)$	70	1	-
$(1s)^2 (2s)$ $(1s)^2 (1d)$ $(1s) (1p)^2$	56	0	+
	56	2	+
	70	0	+
	70	2	+
	20	1	+

are observed experimentally (Fig. 12). Two questions remain open within the framework of this simple model: i) Why do we not see a  $[70, 0^+]$ , a  $[70, 2^+]$ , and a  $[20, 1^+]$  multiplet somewhere in the 1500-2000 MeV region? ii) Why do we observe the radially excited states (the second  $[56, 0^+]$ ) at the same mass region of the first orbital excitation? Both problems hint that the appropriate level structure of the potential is at least slightly shifted with respect to an ideal harmonic oscillator scheme. In fact, all indications are that the  $(1p)$  excited quark is very close to the  $(2s)$  level. In other words, the radially excited states are lower than expected for an oscillator. This conclusion is strengthened by the existence of the  $P_{11}(1750)$  N state which might very well be the first state of the second radially excited  $[56, 0^+]$ , having  $n = 3$ . Various specific models for the quark-quark forces within the symmetric model framework have been proposed in order to explain the observed level sequence. In particular, Mitra<sup>52)</sup> has proposed an S-wave  $qq$  interaction which allows only the  $[56, (2\ell)^+]$  and  $[70, (2\ell+1)^-]$  multiplets.

d) Once we have a "potential" or a "force" which correctly predicts or explains the observed spectrum (including spin dependence, L-S splitting, etc.) we may proceed to compute transition rates<sup>53-55)</sup>. Several such calculations were submitted to this Conference with an over-all fair degree of success. This, however, is the most sensitive aspect of the baryon-spectrum, since small amounts of level mixing which barely

affect the observed masses may induce enormous corrections to the computed transition rates.

### 3.4 Other theoretical schemes for baryons

Various dynamical calculations outside the framework of the quark model lead to interesting predictions for baryon properties.

a) The static model can account for the entire observed baryon spectrum, but predicts "exotic" states such as  $I = 5/2$   $N^*$  and  $S = +1$  states<sup>56)</sup>. It is interesting to note, however, that all the "conventional"  $I = 1/2, 3/2$   $N^*$  resonances which are assigned according to the static model to "exotic" SU(3) representations, were never observed outside the phase-shift analysis. Assuming that the same situation occurs for the  $S = +1$  states, we cannot use the absence of established Z states as evidence against the various static model results.

b) Bootstrap calculations for baryon states, either using the N/D method or the finite-energy-sum-rule idea, have not led to any satisfactory quantitative predictions. The baryon spectrum is much more complicated than the meson spectrum, and it is not surprising that the early quantitative successes of the FESR bootstrap are all related to meson states.

c) The idea of parity-doublet baryon trajectories has had some remarkable successes<sup>57)</sup> but the absence of parity doublets for the  $1/2^+$  octet,  $3/2^+$  decuplet,  $1/2^-$  and  $3/2^-$  singlets, i.e. the four lowest-lying SU(3) multiplets (Fig. 12), has to be explained. One possible explanation is that some dynamical scheme (the quark model?) should tell us when the parity-doublet trajectories materialize into particles, and when their residue functions develop zeros in the right-signature points, and thus avoid making particles. It is a fact, however, that all existing parity doublets correspond to those states in the positive parity 56 ( $L = 0$  or 2) multiplets which have counterparts in the  $[70, 1^-]$ .

d) Chiral SU(2)  $\times$  SU(2) has led to one interesting mass relation<sup>30)</sup>:  $m_\Delta^2 = \cos^2 \theta m_N^2 + \sin^2 \theta m_X^2$ , where  $\theta$  is the mixing angle which indicates how strong are the components of all the  $I = 1/2$  excited N states in the basic  $(1/2, 1)$  SU(2)  $\times$  SU(2) representation which mainly accommodates  $N(940)$  and  $\Delta(1240)$ . The

width  $\Gamma(\Delta \rightarrow N\pi)$  gives  $\cos \theta \sim 0.8$ , and that leads in the above mass formula to  $m_\chi \sim 1650$  MeV, a reasonable value for a weighted average mass of the excited  $I = 1/2$  states which mix with the nucleon.

e) The model of Gell-Mann and Zweig can be extended<sup>19)</sup> to the baryon spectrum, predicting many additional low-lying baryons. Most of these are still missing. The model includes a prescription for answering the question raised above [Section 3.4(c)] concerning the existence of particles on parity doublet trajectories.

#### 4. s-CHANNEL RESONANCES, t-CHANNEL TRAJECTORIES AND LOOPS IN THE ARGAND DIAGRAM

##### 4.1 Resonances in partial wave analysis and Schmid's proposal

The hunt for resonances in the partial wave analysis of various elastic processes is based on the following simple logic:

- A resonance formula, such as the Breit-Wigner expression, leads to an energy variation of the resonating phase shift which can be described by a circle in an Argand diagram for the real and imaginary part of the relevant partial wave amplitude.
- The radius of this circle represents the elasticity of the resonance. The circle can be shifted from the central lowest point of the unitarity circle, if some non-resonating "smooth" background contributions are present.
- Until a short time ago, we did not know of any simple mechanism, other than the Breit-Wigner type of formula, which can produce circles in the Argand plot of a given partial wave.
- It was therefore assumed that every circle (or "any substantial part of a circle") appearing in the Argand plots for the various phase shifts, actually corresponds to a resonance in the same partial wave.

This logic is used extensively by the various phase-shift analysis groups, and a large fraction of the baryon resonances discussed in Section 3 were actually discovered by using this technique.

Several months ago Schmid pointed out<sup>58)</sup> that we do know another simple mechanism of creating circles in the phase-shift Argand diagram. He showed that the expression for the exchange of a Regge trajectory in

the t-channel, when analysed in terms of s-channel partial waves, produces circles in the Argand diagram. These circles strongly resemble the "resonance circles". At this point, if we return to our previous discussion of the "Argand circles", we find that point (c) is not true any more and therefore the conclusion (d) cannot be drawn without a further study of the dynamical meaning of s-channel resonances and t-channel trajectories. But before we proceed (in Section 4.2) to discuss this problem, let us first see how the t-channel Regge trajectories are actually capable of producing such circles. At first glance it seems strange that a function such as  $A(\nu, t) = \xi(\alpha)\nu^{\alpha(t)}$  which has a "smooth" energy dependence can lead to bumps in specific partial wave amplitudes [ $\xi(\alpha)$  includes the signature factor and the necessary "ghost-killing" factors]. It turns out, however, that when the "smooth" curve describing  $\text{Im } A(\nu)$  as a function of  $\nu$  is decomposed into its partial waves, every one of them peaks at a different set of energy values and they all combine to give the "bumpless"  $\nu^{\alpha(t)}$  form. Such a situation is schematically illustrated in Fig. 13.

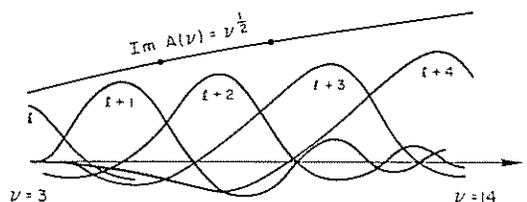


Fig. 13 A typical decomposition of an amplitude of the form  $A(\nu, t) = \xi(\alpha)(\nu/\nu_0)^{\alpha(t)}$  into its various partial wave contributions, for  $\alpha(t) = 1/2 + t$ .  $\text{Im } A(\nu, 0)$  is plotted against  $\nu$  in the range  $3 \leq \nu \leq 14$  in units of  $\nu_0$ .

A simple way of understanding how this type of decomposition comes about is the following<sup>59)</sup>: consider an elastic scattering process of equal mass particles with an amplitude of the form  $A(\nu, t) = e^{i\pi\alpha(t)}$ , where  $\alpha(t) = a + bt$ . At any given energy the angular dependence of both  $\text{Re } A(\nu, t)$  and  $\text{Im } A(\nu, t)$  will be given by curves similar to those of Fig. 14. In order to extract (for example) the S-wave projection of this amplitude at a given energy, we simply have to integrate the curves of Fig. 14 over the interval from  $\theta = 0^\circ$  to  $180^\circ$ . Since  $\theta = 180^\circ$  corresponds to different values of  $t$  at different energies, the integration range will increase continuously with energy,

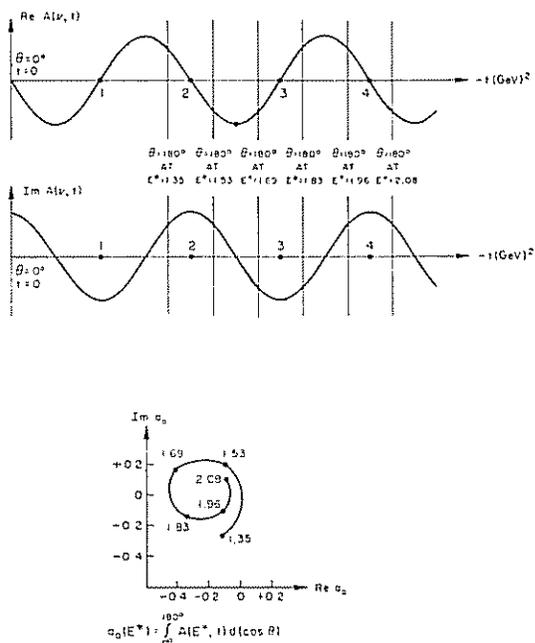


Fig. 14  $\text{Re } A(v, t)$  and  $\text{Im } A(v, t)$  plotted against  $t$ .  $A(v, t) = e^{i\pi\alpha(t)}$ ;  $\alpha(t) = 0.5 + t$ . The  $\theta = 180^\circ$   $t$ -values at various c.m. energies  $E^*$  are marked, assuming that the process under discussion is elastic  $\pi\pi$  scattering. The S-wave projection of  $A(v, t)$  is plotted below as a function of  $E^*$  in an Argand plot.

and both real and imaginary parts of the S-wave projection will oscillate between zero and  $1/2\pi b q^2$  (where  $q$  is the centre-of-mass momentum). We should therefore expect the S-wave phase shift to describe in the Argand plot a spiral with a radius which decreases like  $1/v$ . Similar considerations hold for all other partial waves. A typical Argand plot for the  $\ell = 1$  projection of the above amplitude is given in Fig. 15.

The full Regge amplitude is, of course, more complicated than our  $e^{i\pi\alpha(t)}$  example. The other terms can change the relative size of the circles, their position, and the frequency of their occurrence<sup>59)</sup>, but the basic feature of a "spiraling phase shift" in the Argand diagram would remain in almost all cases.

We have therefore learned from Schmid's work that the Argand circles can result from the exchange of Regge trajectories in another channel, as well as from ordinary s-channel resonances. How do we interpret a circle which is actually found in some phase-shift analysis?

4.2 The interpretation of circles in the Argand plot

Every scattering amplitude in the physical region can, in principle, be expanded in terms of partial waves in the s-channel, and represented as a sum of many s-channel resonances and a background contribution. Presumably, the same amplitude can also be described in terms of a sufficiently large number of t-channel Regge poles, cuts, a background integral, and perhaps other types of t-channel singularities (fixed poles?). Every one of these two descriptions is a complete parametrization of the physical amplitudes. Dolen, Horn and Schmid<sup>60)</sup> have shown that it would be erroneous to assume that the correspondence between these two descriptions is such that the leading trajectories in the t-channel are always included in the s-channel background. They argue that the s-channel resonances are actually "building" at least part (and in some cases, most) of the contribution of the leading t-channel trajectories. Our perturbation theory intuition, which is implicitly based on secretly drawing Feynman diagrams for strong interaction processes, tells us that we could add s-channel resonances and t-channel exchanges<sup>61)</sup>. This is now known to be generally incorrect<sup>60)</sup>. It may work in a few cases but, in general, severe double counting is involved.

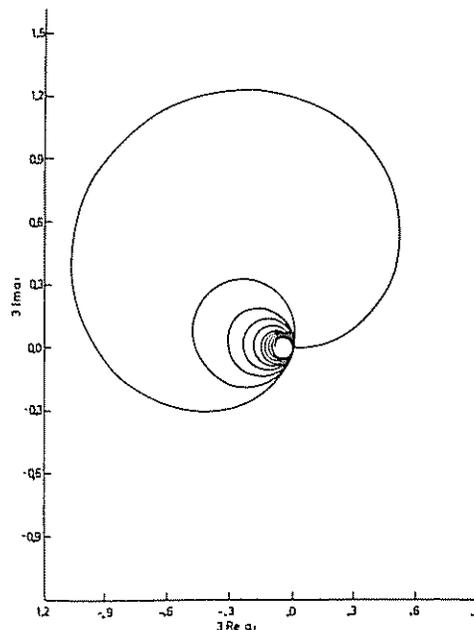


Fig. 15 The P-wave Argand plot of  $A(v, t) = e^{i\pi\alpha(t)}$ . The figure is taken from Chiu and Kotanski, Ref. 59.

All of these remarks lead us to the principle that we will use in interpreting the circles in the Argand diagram. It is perfectly possible that at a given energy region, a physical scattering amplitude can be approximately described either by a few s-channel resonances or by a few t-channel Regge trajectories. Both descriptions will be complementary in such a case, and any one of them would be an adequate interpretation of the data. Note that we do not actually claim that this is always the case. From a purely mathematical point of view, we can probably always parametrize the amplitudes in terms of an arbitrarily complicated sum of resonances or an arbitrarily horrible combination of Regge poles. However, the real issue here is the simplicity of such descriptions.

We can envisage four different situations (with "smooth" transition regions between them):

- a) In some cases the s-channel resonance description is simple whilst the t-channel characterization is complicated and useless. It is clear, for example, that  $\pi N$  scattering at, say, 1200-1300 MeV c.m. energy can be described simply in terms of s-channel resonances (one of them is sufficient, in fact), whilst a t-channel description of the scattering in this region would require an extremely awkward and highly artificial collection of exchange terms.
- b) The reverse situation occurs, for example, in  $\pi N$  charge exchange scattering at 10-20 GeV, which can be simply parametrized in terms of t-channel trajectories, whilst an s-channel resonance description would, at best, involve a huge number of overlapping inelastic resonances.
- c) There could be cases in which both descriptions are complicated.
- d) Finally there may exist regions in which both pictures are relatively simple.

The last type of situation is the most interesting from the point of view of strong interaction dynamics, since the consistency requirements between the s-channel and t-channel descriptions are essentially bootstrap-type equations. This possibility is discussed elsewhere in these proceedings<sup>6,2)</sup>.

Here we will utilize this analysis only for the purpose of understanding the Argand circles. The

philosophy outlined above, essentially tells us that we should continue to interpret the circles found in the phase-shift analysis as resonances. In some or all cases, the same circles may be alternatively generated by t-channel trajectories, but that does not contradict our resonance interpretation, in view of the principle which tells us that the two descriptions can peacefully coexist.

One might wonder whether we have forgotten that a resonance should always be accompanied by a pole in the unphysical region, on the second sheet, and that this is a unique, well-defined, characterization of a resonance. We do not ignore this point. We would like, however, to make two remarks connected to the question of the existence of such poles.

1) It has been shown<sup>17,6,3)</sup> that it is possible to build mathematical examples which have asymptotic Regge behaviour in s, and s-channel resonance poles. This means that, in principle, we may assume that every Argand circle does correspond to a second-sheet pole, but that at the same time it can be described by a typical Regge term.

2) In principle, because of the finite experimental errors, we will never be able to continue analytically the physical amplitude to the unphysical region in a unique way, and therefore will never be able to prove beyond any doubt the existence of the resonance pole. This remark, which should normally be of a purely academic nature, is relevant here in view of the possibility of constructing the following example. Given a physical scattering amplitude in a given section of the physical region, and a pre-assigned degree of accuracy, one can always build two different representations which will both reproduce the physical amplitude within the required degree of accuracy. One representation would have the second-sheet poles (and will essentially be a sum of Breit-Wigner-type expressions) and the other will not possess such poles and will be of the form

$$\sum_i \xi(\alpha_i) v^{\alpha_i(t)} .$$

If both representations are simple there is no harm in using either one of them. In other words, there is nothing wrong with assuming that the observed

Argand circles are both resonances and the products of t-channel trajectories, and the "unique" definition of a resonance as a second-sheet pole is not going to resolve this dual nature of the amplitude.

#### 4.3 The properties of a resonance

If we accept the interpretation outlined above, we will have to demand that resonances obey all the usual requirements of a physical state:

- a) A resonant state should have a well-defined set of quantum numbers -- spin, isospin, etc.
- b) The same resonances with the same characteristics (mass and total width) should appear in all channels which can couple to a particular set of quantum numbers.
- c) An s-channel resonance should obey factorization in the s-channel.
- d) Every resonance should, in principle, be produced in production experiments in which it would appear in the final state together with other particles.

These are non-trivial requirements for an object which can be alternatively described as a sum of t-channel trajectories!

There are two different points of view that one can take with respect to these requirements:

- i) One may simply state that an expression based on simple t-channel properties cannot satisfy such requirements, and therefore the correspondence between s-channel resonances and t-channel trajectories cannot hold.
- ii) One may interpret requirements (a)-(d) as further restrictions imposed on the dynamical equations which emerge from the consistency between the two complete descriptions of the amplitude.

As long as the second "optimistic" point of view has not been proved wrong, we prefer to accept it and to hope that such restrictions actually lead to relations between the parameters in the different channels. (See also Section 5.2.)

Now that we have discussed the more philosophical aspects of this interesting question, we should return to the real world and ask ourselves: How useful is this picture, and do we actually have regions in

which the s-channel and the t-channel descriptions are simultaneously simple?

#### 4.4 Applications, tests, and general remarks

a) Schmid, in his original paper<sup>63)</sup>, actually presented an approximate calculation of a few s-channel partial-wave projections of the t-channel  $\rho$ -trajectory in  $\pi N$  scattering. He produced the correct positions of some of the high-mass  $N^*$  states. Collins, Johnson and Squires<sup>64)</sup> carried the analysis in a more quantitative way down to much lower energies and obtained some results in remarkable agreement with the actual "experimental" phase shifts, and some which do not resemble them at all. The successful results are displayed in Fig. 16, but we must add that the failures are equally impressive. Kreps and Logan<sup>65)</sup> performed a calculation similar to that of Schmid, but omitted some of his approximations and used a different high-energy parametrization for the  $\rho$  residue. Their results are entirely different from his.

As far as we can see, there are two lessons to be learned here:

- i) The precise form of a given partial wave projection of a t-channel trajectory may depend on relatively minor details of the t-channel parametrization (which is always derived from the high-energy data).

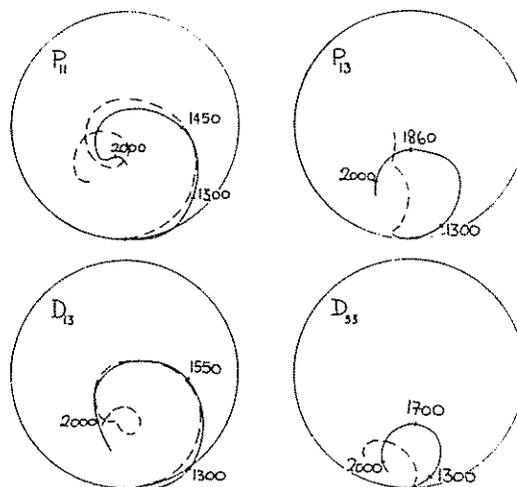


Fig. 16 Argand diagrams for partial waves in  $\pi N$  scattering in which the projected t- and u-channel trajectories correspond most closely to the "experimental" phase shifts. The full lines are the trajectory projections and the dashed lines are the phase shifts of Ref. 37. The figure is taken from Collins et al., Ref. 64.

This is especially correct at the lower energy region, where even the small backward peak becomes crucial<sup>64)</sup>.

ii) One should always try to separate those aspects of the analysis which are relatively independent of the unknown details of the high-energy data, from those which depend crucially on a specific detail. Features of the first type should be used in order to test the general ideas which are under discussion here. Those of the second type may be perhaps used in resolving ambiguities in the high-energy parametrization, once the general idea is proved successful.

A detailed analysis of this kind is clearly needed before we can decide whether or not the correspondence between the two channels can be a useful concept.

b) Alessandrini and Squires<sup>66)</sup> pointed out that the resonances appearing in  $\pi N \rightarrow \pi N$  should also appear in the reaction  $\pi^- p \rightarrow K^+ \Sigma^-$ , in which no (doubly charged) trajectory is known to be exchanged. They considered this an argument against the relevance of t-channel exchanges at the resonance region. It seems to me, however, that the absence of  $I = 3/2$  t-channel trajectories merely predicts here that the  $I = 1/2$  and  $I = 3/2$   $N^*$  resonances in  $\pi^- p \rightarrow K^+ \Sigma^-$  should approximately cancel each other, at least in some average sense. It will be interesting to find out whether this is confirmed by future  $\pi N \rightarrow K \Sigma$  phase-shift analysis.

c) Rubinstein, Schwimmer, Veneziano and Virasoro<sup>16)</sup> considered the s-channel partial wave projections of the t-channel  $\rho$ -exchange in the reaction  $\pi \pi \rightarrow \pi \omega$ . They found that not only the resonances on the  $\rho$  trajectory are produced in the Argand plots, but also additional states which may be interpreted as building a set of parallel secondary trajectories (daughters?). This cannot be tested experimentally at present, but the general structure of the  $N$  and  $\Delta$  spectra hints that the result may be true (see Section 4.5).

d) Chew and Pignotti<sup>67)</sup> pointed out that in the same way that s-channel resonances are alternatively described as combinations of t-channel trajectories, one can describe a resonance-production process such as  $\pi + N \rightarrow A_1 + N$  as a sum of multi-Regge exchanges in  $\pi + N \rightarrow \pi + \rho + N$ . The main moral here is that

we should not add the  $A_1$  production amplitude to the multiparticle exchange diagram known as the "Deck mechanism". This would involve the same type of double counting as the one committed by the interference model<sup>61)</sup>. It is therefore a priori incorrect to subtract the "Deck background" from the  $\pi + N \rightarrow \pi + \rho + N$  amplitude and then look for the  $A_1$  enhancement. The subtracted part may include a large portion of the  $A_1$  itself!

e) It has been argued<sup>68)</sup> that the  $Z_1$  enhancement in  $\sigma_t(K^+ p)$  around 1900 MeV follows from a sharp rise at threshold of  $\sigma(K^+ p \rightarrow K \Delta)$ , and that this sharp rise can be accounted for by some kind of a  $\rho$ -exchange mechanism. Within the framework of the dual description of amplitudes, this argument cannot be considered as evidence against the existence of the  $Z_1$  state!

f) From the point of view of the quark model, it is extremely hard to understand how real physical quarks could build resonance states that satisfy such inter-channel conditions. If, however, the quark model merely reflects some deeper dynamics and some algebraic order among the resonant states, no such difficulties should emerge here.

#### 4.5 Are the t-channel trajectories reflected in the observed meson and baryon spectrum?

A quantitative answer to this question has not been given so far, and much more careful work will have to be done before any conclusions are reached. Some qualitative remarks should be made, however:

a) The apparent absence of "exotic" mesons and baryons corresponds, from the t-channel point of view, to cancellations between the contributing trajectories. These cancellations are precisely those of the "exchange degeneracy" picture<sup>69)</sup>.

b) Since the exchange degeneracy relations are only approximately true, it is likely that "small" exotic resonances, such as the  $Z$  states, will be found.

c) Whilst the absence of prominent states with a given set of internal quantum numbers is easy to interpret, it is very hard to imagine that states with specific spin-parity assignments will be consistently absent. In fact, the simplest conclusion that we can draw from the t-channel picture is that every partial

wave amplitude should sooner or later show circles in its Argand plot. Whilst it is not entirely impossible, we find it hard to see how a certain  $J^P$  sequence can be forbidden, whilst others are allowed. Such considerations lead us to the following conclusions:

i) Mesons with all possible  $J^{PC}$  values should exist. This supports the new Gell-Mann - Zweig proposal<sup>19)</sup> (Section 2.2) and contradicts the "old" quark model.

ii) Every single partial wave should resonate in the baryon spectrum. Figures 5 to 8 demonstrate clearly that this is actually the case.

iii) Whilst all  $J^P$  values should eventually correspond to resonances, there could be some  $J^P$  sequences with more prominent resonances than others in a channel with a given set of internal quantum numbers. Chiu and Kotanski<sup>59)</sup> have shown, for example, that the prominent  $N^*$  states should obey the relation  $J-L = I - 1$ , if the t-channel representation is to be trusted. This is experimentally verified:  $N(940)$ ,  $\Delta(1240)$ ,  $N(1520)$ ,  $N(1690, 5/2^+)$ , and  $\Delta(1950)$  are all  $J-L = I-1$  states!

d) In any given partial wave amplitude we should expect a sequence of resonances with decreasing elasticities and more or less equal spacing in mass (or squared-mass). This is implied by the general features of the partial-wave-analysed Regge amplitude (see, for example, Fig. 15). The  $N$  and  $\Delta$  spectrum, which are known up to 2 GeV (Figs. 5 and 6) indeed show that all the partial waves that resonate below 1600 MeV exhibit at least one more resonance candidate at a higher mass. From this point of view, we should have been worried if the  $N(1470, 1/2^+)$  state did not exist! (Contrary to the usual attitude of theorists who always considered this state as an undesirable feature of our world ...)

We offer these speculative remarks only as an indication that this approach is worth studying, and not as an attempt to summarize its predictions. This entire concept, which was developed only in the last few months, certainly deserves a lot of attention on the part of both theorists and experimentalists who are more directly involved in the actual search and interpretation of new resonant states.

## 5. A WORLD OF RESONANCES

### 5.1 Is everything made out of resonances?

We close this report with a short discussion of an intriguing possibility which has perhaps become a little more realistic in the last year or two.

Can we describe the entire world of strong interaction processes in terms of resonances only?

If we have an infinite number of secondary trajectories, and if all of them are infinitely rising, we should not be surprised if scattering amplitudes at, say, 50 GeV can be described in terms of a huge number of s-channel resonances. Moreover, if we look at the many invariant mass plots included, for example, in Dr. French's report to this Conference<sup>1)</sup> we find that an imaginative theorist could interpret every one of them as a sum of resonance bumps with very little or no background. We know that the  $\rho\pi$  or  $K^*\pi$  invariant mass plots can be mostly accounted for if we believe that  $A_1$ ,  $A_{1.5}$ ,  $A_2^L$ ,  $A_2^H$ ,  $A_3$ ,  $K^*(1230)$ ,  $K^*(1320)$ ,  $K^*(1420)$ ,  $L$ , and perhaps  $H$  and  $K^*(1280)$ , exist. But even the  $\pi\pi$  and  $K\pi$  invariant mass plots can be shown to include very little "meat" in addition to the  $\sigma$ ,  $\rho$ ,  $S^*$ ,  $f^0$ ,  $g$  mesons in  $\pi\pi$ , and the  $K^*(890)$ ,  $K^*(1100, 0^+)$ , and  $K^*(1420)$  in  $K\pi$ . If this is a general feature, and it may very well be, we are approaching a situation in which

i) all two-body final-state processes will be described in terms of many direct-channel resonances or t-channel trajectories (for an exception, see Section 5.2);

ii) all processes with three-body final states will be described either as a sum of quasi-two-body final states or as a sum of multiperipheral exchanges;

iii) the quasi-two-body description of three-body final states will itself have the features mentioned in point (i) above.

The most extreme point of view that one could take is illustrated in Fig. 17, where every one of the schematic diagrams is supposed to give by itself a complete description of the process  $a + b + c + d + e$ . This is a gross oversimplification of the situation, and it certainly cannot be true in this naive form. Moreover, even if it were true, it would

have been entirely useless in most cases. We bring it here only in order to provoke some thought in this direction in view of the accumulating experimental evidence for the multiresonance-dominance assumption, and the theoretical developments described in Section 4.

### 5.2 An exception: the Pomeranchon

One outstanding exception to the above speculation is the Pomeranchon, or the simple optical diffraction mechanism. This type of contribution to elastic and quasi-elastic scattering amplitudes seems to be outside the domain of resonance dominance<sup>70)</sup>. We will not discuss this in detail here, but simply point out that processes in which s-channel resonances are absent (e.g.  $K^+p$ ,  $pp$ , or  $\pi^+\pi^+$  elastic scattering or the invariant mass plots of the same particles) do have as large a Pomeranchon contribution as do processes with a rich resonance spectrum (such as  $K^-p$ ,  $\bar{p}p$ ,  $\pi^+\pi^-$ ). This implies that the t-channel Pomeranchon corresponds to some isospin-independent background in the direct channel<sup>70)</sup>. This conjecture leads to a large number of interesting predictions which are all consistent with (and sometimes in remarkable agreement with) experiment<sup>70, 71)</sup>.

### 5.3 Infinitely rising trajectories: a few remarks

We close our discussion of a world "made of resonances" with a few remarks on the subject of infinitely rising trajectories.

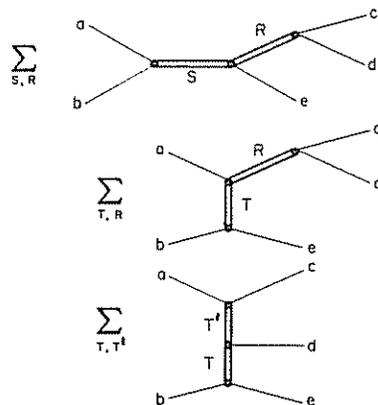


Fig. 17 Schematic diagrams for various possible descriptions of the process  $a + b + c + d + e$ . Every diagram is supposed to represent (when summed over all possible intermediate single particle states) a complete description of the reaction in an extreme model in which all channels are dominated by sums of resonances (or trajectories).

a) Experimentally there are strong indications for linearly rising trajectories, both in the  $N^*$  spectrum and in the meson spectrum (R-S-T-U, etc.). Whether this trend will continue, or will gradually change into some other type of energy-dependence of the trajectories, remains to be seen. The boson spectrometer experiments are probably the easiest way of studying this problem.

b) All bootstrap-type models which have been suggested so far and which have Regge behaviour in all channels and resonances in all channels, involve linearly rising trajectories. We do not know whether this is a necessary feature in such a model, and it would be interesting to analyse this problem.

c) The simple optical "absorption" picture suggests that inelastic processes are dominated at a given c.m. momentum  $k$  by a "ring" of partial waves having  $\ell_{\min} \leq \ell \leq \ell_{\max}$ , where  $\ell_{\min}$  and  $\ell_{\max}$  are proportional to  $k \propto \sqrt{s}$ . From the point of view of s-channel trajectories, this would mean that most of the "strength" of the amplitudes would come from a chain of resonances obeying  $\ell \propto \sqrt{s}$ . This could happen if: i) the trajectories rise like  $\sqrt{s}$ ; ii) the trajectories are linear but the "important" states on the trajectories lie on a  $\sqrt{s}$ -type curve.

d) The same conclusion with respect to a  $\sqrt{s}$  behaviour of the contributing s-channel resonances can be reached by high-energy s-channel phase-shift analysis of an ordinary t-channel trajectory. The same two possibilities as in (c) exist<sup>72)</sup>.

e) If the trajectories are linear, low partial wave decays of the higher states on the leading trajectories will become more and more difficult, since the spacing in mass ( $\propto \sqrt{s}$ ) between two adjacent states tends to zero. If low- $\ell$  decays are energetically forbidden and large- $\ell$  decays are weak because of angular momentum barriers, the high-mass states on the leading trajectories may become narrower and narrower<sup>73)</sup>.

The discovery of high-mass states and the determination of the trajectory form at high energy should be an extremely interesting challenge to experimentalists.

## 6. SUMMARY

I would like to conclude by presenting an (obviously biased) brief summary of the main theoretical aspects of the hadronic resonance spectrum which were discussed in this Conference:

- 1) SU(3) symmetry emerged, again, as an extremely powerful classification scheme, in view of the recent discovery of many of the missing hyperon resonances. The applicability of SU(3) predictions to transition rates remains limited, due to complicated mixing effects.
- 2) The quark-model classification of mesons remains successful, but the theoretical inconsistency with a parallel-daughter Regge picture deserves attention. An experimental search for natural parity states with abnormal charge conjugation is desired.

3) The quark model description of the baryon spectrum is extremely successful. Among the various versions, the symmetric model is the most attractive.

4) The new dynamical interpretation of Argand diagram circles as projections of t-channel exchange mechanisms has had some success. More quantitative tests of this extremely interesting theoretical idea are badly needed.

5) The description of scattering amplitudes in terms of sums of resonances is at least partly supported by the data. A careful theoretical analysis of this question is especially important in view of its immediate connection with the experimental resonance-searching techniques.

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## DISCUSSION

GOOD: Why do octet t-channel exchanges not lead to unobserved s-channel resonances,  $\bar{10}$ 's for example; and similarly why do the observed s-channel octets and decuplets not lead to (unobserved)  $27$ 's in the t-channel?

HARARI: If somebody would have given me the statement that in the s-channel there were only singlet and octet mesons and asked me to predict what would happen in the t-channel, I am not sure that I would know what to answer, except that I would expect the t-channel and s-channel to resonate in the same quantum numbers. But there is an entirely different point of view that you can take. You can ask yourself what is the minimal set of quantum numbers that is allowed, such that you exchange only this set of quantum numbers in the s-channel and only this set in the t-channel and remain self-consistent. For example, if you have only SU(3) singlets in the world you immediately get into contradictions. Suppose you have s-channel singlet mesons, then you cannot have only t-channel singlet mesons, while if you have singlets and octets you are all right. It turns out that singlets and octets are the simplest system of mesons, which is self-consistent in terms of s-channel and t-channel exchanges. It is not the only system, but it is the simplest. And if you really want to speculate wildly, you could even invent a principle which says that the entire system of the allowed quantum numbers for mesons or for baryons is the minimal system, which should be self-consistent, between the s- and the t-channel in this duality picture. If you make this assumption, you actually predict that there will be only singlet and octet mesons and only singlet, octet, and decuplet baryons. (You cannot distinguish between decuplets and anti-decuplets.) These are the minimal systems. Why there should be such a principle, I have no idea.

BREIT: Resonances have been under discussion in low-energy nuclear physics for some years, and there are some similarities as well as differences of viewpoint from those we just heard. Regge trajectories have not been used much in nuclear physics however. There is complete agreement about the maximum of the

total cross-section not furnishing an adequate criterion of the resonance position and the necessity of using partial waves, as brought out by the speaker. The  $90^\circ$  phase shift is often not the best criterion. A better one is the energy at which the phase shift varies most rapidly, as may easily be seen by superposing a small external field, such as a Coulomb field, on the existing ones. Doing so also shows that singularities in the complex plane can obscure at times the principal simple features of a physical system. Thus a relatively insignificant field can introduce essential singularities. Removal of unimportant interactions and dealing with a simplified model is therefore a useful device. The representation of scattering amplitudes directly by a linear superposition of ordinary single-level resonance terms is not possible in general, but it is possible indirectly as in the Kapur-Peierls and the Wigner R-matrix theories. The transition to Regge poles can be made from the latter as shown by Wigner and, in more detail, by Roskies. But such representations are not unique as may be seen particularly clearly in Wigner's R-matrix theory, which gives an infinite number of representations as the "nuclear radius" is varied. These matters require much discussion but in view of the lateness of the hour it appears best to stop.

FREUND: In answer to Dr. Good's question, I would like to point out that the dynamical principle that makes duality possible with only meson nonets and baryon octets, decimets and singlets, is exchange degeneracy. Take for instance  $K^+p$  elastic scattering. Here both tensor and vector nonet trajectories can contribute. If exchange degeneracy holds, the two contributions will exactly cancel in the imaginary part of the  $K^+p$  amplitude. By duality this will forbid the existence of resonances in the  $K^+p$  channel.

LINDENBAUM: I agree with Harari and have for many years felt that particle production proceeds predominantly, and perhaps almost exclusively, by formation of excited states or isobars which then decay.

I certainly also agree with the equivalence of parametrizations of strong interaction by either s-

channel resonances or t-channel poles. However, when one tries to represent strong interactions at high energies by t-channel singularities, one finds that in addition to an ever-increasing number of resonances discovered (which are then represented by t-channel poles) even additional non-discovered poles are continually required to fit the data. Furthermore, the leading singularity at high energies acts like a cut, or if you prefer a mysteriously flat Pomeranchuk trajectory. I believe this latter effect is just an equivalent t-channel representation for constant or near constant total cross-section behaviour at high energies -- a conjecture made a long time ago and which is very consistent with and implied by all present data.

Those attempts to parametrize strong interactions by a series of Regge poles and other t-channel singularities have led to an excessive and still increasing number of parameters.

Although the parametrizations may be of phenomenological and calculational value, I would not take them very seriously until a dynamical theory or even a model based on a few stable parameters is obtained.

HARARI: Of course, every one of us agrees with Professor Lindenbaum that a theory with a small number of parameters is badly needed. That goes beyond any discussion.

Concerning the Pomeranchon, I did not want to discuss it in my talk, because it is not really very much related, but all the indications are that the Pomeranchuk-trajectory is really a reflection -- as far as I can see it -- of some kind of diffraction scattering, which has nothing to do with the resonances either in the s-channel or the t-channel, and is a quantity that has to be added onto the contributions of s-channel or t-channel particles in every elastic or quasi-elastic process. This is presumably something which is outside the duality picture -- as far as I can see it. Other possibilities exist, but this is my point of view.

BUCCELLA: The  $SU(6) \otimes O(3)$  classification scheme may be obtained from:

- i)  $SU(3) \otimes SU(3)$  chiral algebra;
- ii) the axial charges having  $\Delta J = 1$  in the  $P_2 = \infty$  reference system.

This second assumption is not rigorously experimentally verified, but it holds rather well, since the nucleon and the  $\bar{3}$ - $\bar{3}$  resonance give the largest contributions to the Adler-Weisberger sum rule.

So the  $SU(6) \otimes O(3)$  classification may be derived without any need of quarks.

ZWEIG: The Deck effect is kinematic in the sense that it does not depend upon the size of the coupling constant; that is, even if the coupling constants involved were very small you would still generate bumps in invariant mass plots. On the other hand, you feel you would have resonances only in strong coupling theory. Does not this discrepancy make the duality concept slightly weaker?

HARARI: I do not know if the duality picture can be extended into non-strong interactions. It seems to me that it will not be extended, because it contradicts the simple Feynman diagram picture in which you do add the t-channel and s-channel exchanges. I only say: you just have to be careful. Once you have proved that in some cases t-channel plus s-channel addition is bad, you should be careful not to do it without thinking. That is the only point I was making. And that is why I say, do not subtract the Deck-effect from the data and then search for the  $A_1$ . This is bad in principle.

DULLEMOND: I want to make a remark pertaining to the last two talks. As things stand now, it is possible to reproduce the established baryon resonances [ $SU(3)$  assignment, spin, parity] together with their approximate mass values, with the help of a simple static strong coupling model, if  $\ell$ -excitations (or radial excitations) are properly built in. No quarks are required.