ANCIENT AND CLASSIC WORLS

Until the 5-th Century

ASTRONOMY

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Weizmann Institute, Israel

What is the significance of Astronomy in antiquity ?

The unreachable sky fascinated humans, and connected to mystical beliefs and religion.

The cycles of day/night of the sun, moon phases, and stars movements were a puzzle, but also created a feel of peace & order in the universe. This secure feel was therefore broken by non-periodical events such as the eclipse (sun and moon), tides, floods, storms and lightening. “The flood” is a reoccurring theme in ancient mythologies.

The seasons of the year had to be anticipated for agricultural preparations (plowing and seeding). They were related to the position of stars at midnight or just before sunset.

The north star offered a fixed (north) direction for convoys and sailors. Its angle was later related to coordinates on the map.

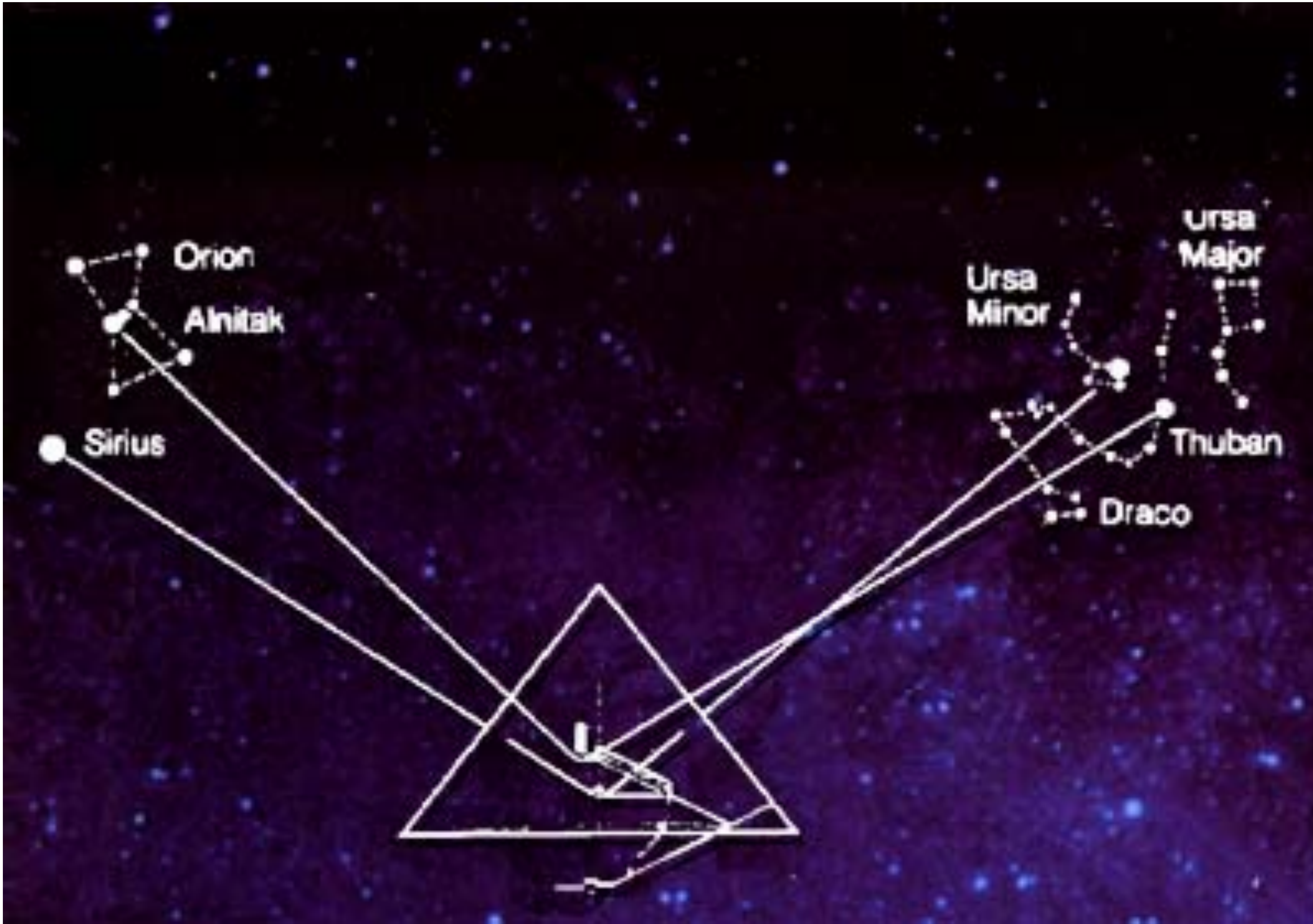
The first astronomical studies were conducted in Babylon and Egypt. They motivated development of mathematics and experimental measurement tools. The 60-base number counting system in Babylon was probably related to astronomical measurements (and the factorization into 1,2,3,4,5,6 that facilitated fractional calculations, and set 12 month 60 seconds/minute, 60 minutes/hour & 360 degrees)

Ancient observation of star movements already recognized the migration of a few stars (the planets) against the stable position of other stars. The early astronomical models therefore claimed that the sky is not made of one solid entity, but contains several “spheres”.

Ancient “astronomical observatories”: similar structures around the world.

In Egypt and in central America (Aztec and Maya) large efforts were invested to build huge structures that served pivots for sun or star directions and helped determine the yearly calendar:

Orientation of the corridor to a pyramid burial room is aligned once a year with the star associated with the buried king.



The “sun and moon” Aztec pyramids.
Were probably aligned with sunset direction twice a year

Maya star observatory. Determined the agricultural season by the direction of the sunset with respect to openings in the building.

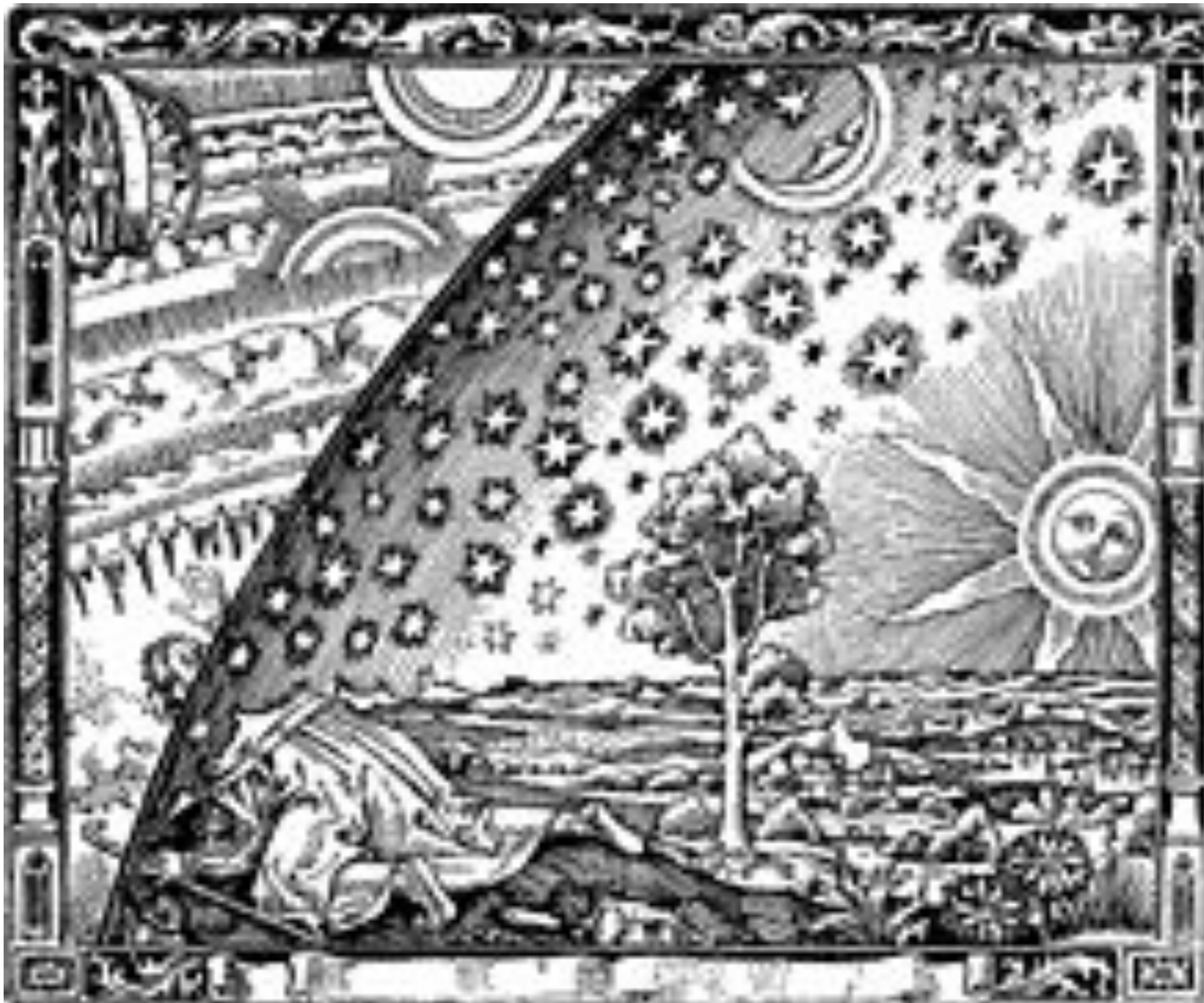
Astronomy and Mathematics developed side-by-side

Since the sky "rotates" around us, star positions are relative to each other.
Measurement of angles between star positions depend on **geometry and trigonometry.**

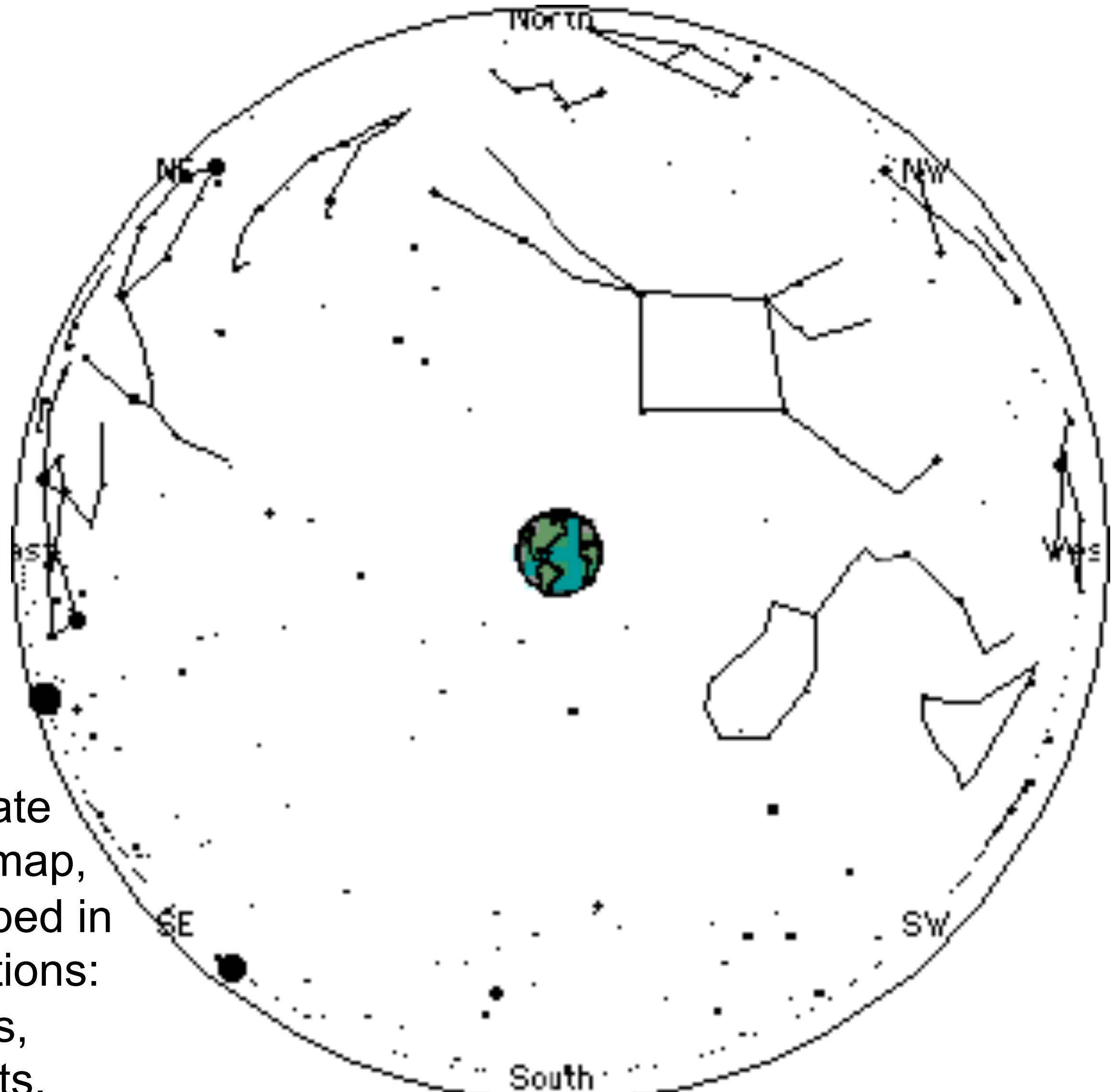
625-547 BC Thales of Miletus – Predicted eclipse by understanding the periodic motion of the sun and the moon.
569-475 BC - Pythagoras of Samos – Believed that the earth is at the center of the universe. He measured the tilt angle of the moon orbit with respect to the equatorial. He observed that the evening Venus is the same Venus rising at the next morning.
Since we see only "half" of the sky scene around us at any time, Pythagoras gave an estimate to the number of stars from counting the number in a small segment of the sky, and scaling for the entire sky.
Would you accept this calculation? Is density of stars uniform across the sky?
There are only about 1000 stars visible by eye, including nebulae.
With telescopes, the number depends on the size of the telescope: larger collects more light to detect less bright stars.

Oenopides of Chios ~490-420 BC – measured the angle between the equator and the sun orbit to be 24^o.
Anaxagoras of Clazomenae 500-428 BC – The moon gets its light from the sun. He explains sun and moon eclipses.

An ancient view of the universe. The earth is flat, and if you go far you will hit the sky dome. Stars are holes in this dome, the sun and the moon circles in front of the dome.

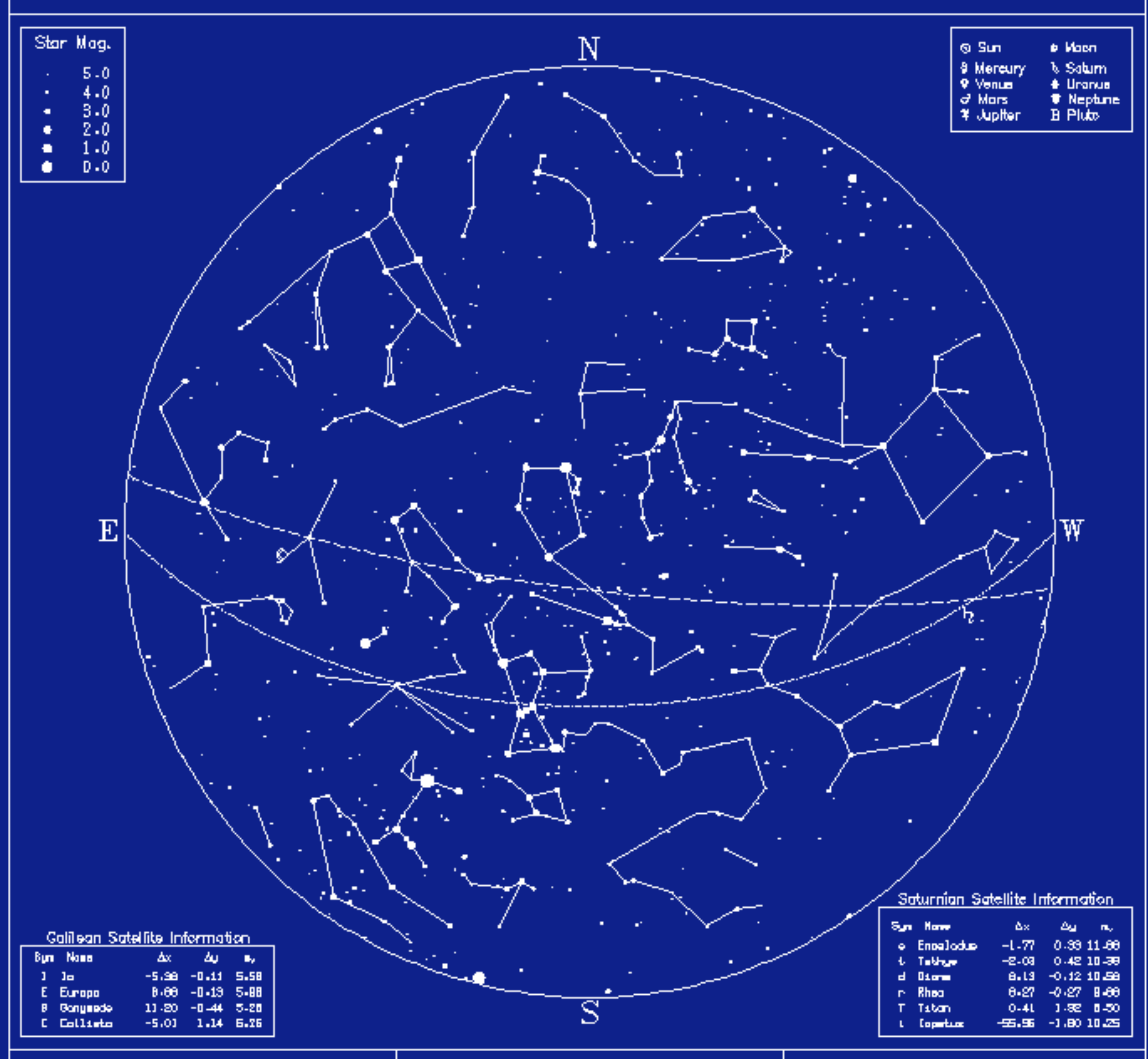


The sky seems to us as a rotating dome. Even on time scale of centuries relative positions of stars (other than the planets) does not vary. The rotation axis pass through the north star. Its angle from the horizon vary from one place to another on earth, and depend on season at each position. It can help determine the longitude on the globe.



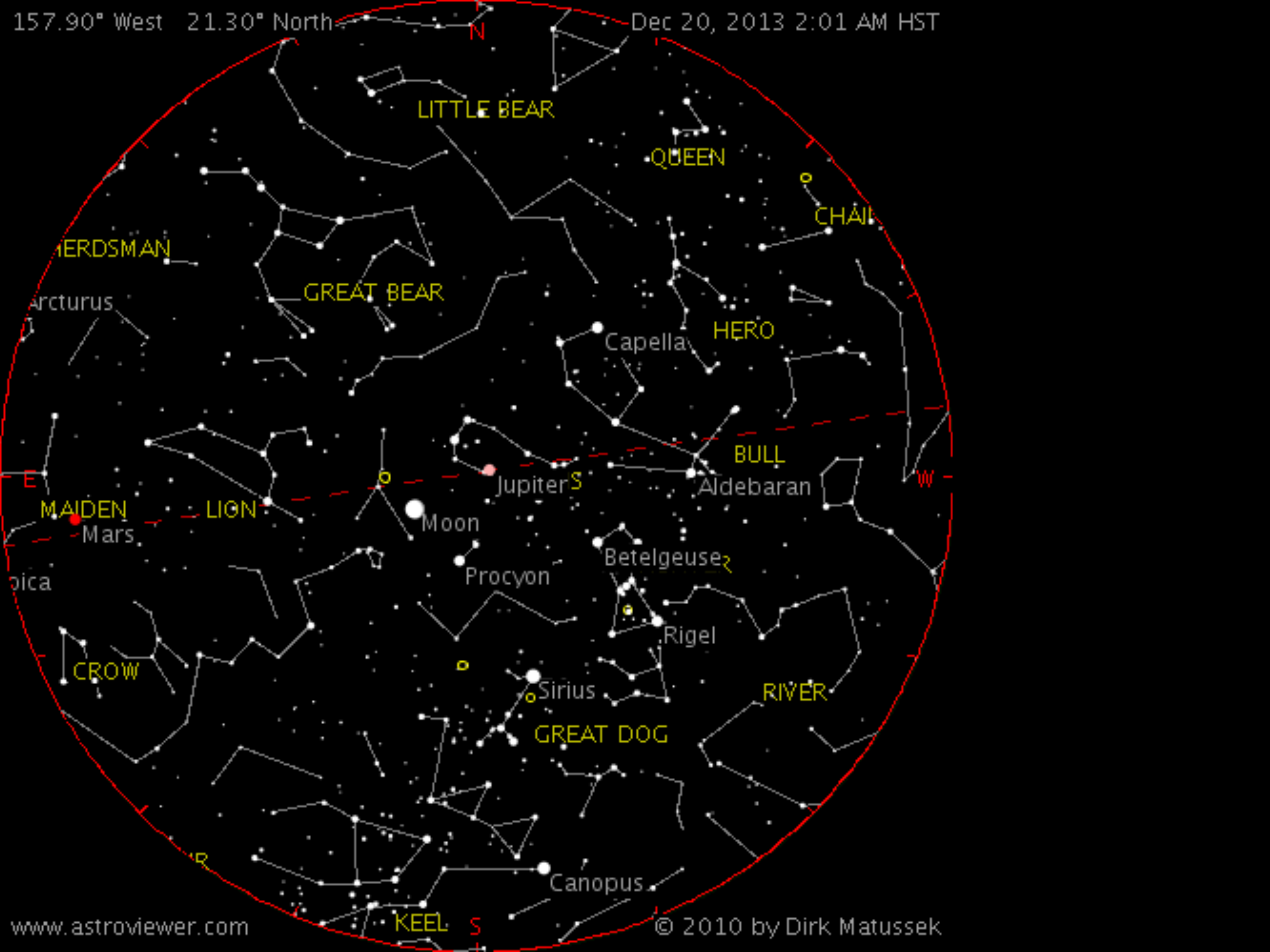
In order to facilitate reading the sky map, strong stars shaped in imaginary formations: the constellations, are used as pivots.

The Zodiac assigns the 12 constellations to 12 months.



Map of the Constellations In the sky.
Only "half" of The 4π spherical angles of the sky can be seen at any moment.
In the following slides different parts, orientations and projections of the sky map are presented.


Pegasus
Cetus
Mira
Aquarius
Taurus
Fomalhaut
Altair
Phoenix
Eridanus
Grus
Capricornus
Aquila
Achernar
Tucana
Orion
Rigel
Dorado
Hydrus
Pavo
Sagittarius
Columba
Serpens Cauda
Lepus
Ara
Betelgeuse
Canopus
Volans
Apus
Carina
Musca
TriA
Sirius
Rigil Kent
Ophiuchus
Canis Major
Antares
Vela
Crux
Lupus
Scorpius
Centaurus
Procyon
Canis Minor
Libra
Serpens Caput
Hydra
Corvus
Spica
Virgo


157.90° West 21.30° North
Dec 20, 2013 2:01 AM HST
N
LITTLE BEAR
QUEEN
CHAI
HERDSMAN
GREAT BEAR
Arcturus
Capella
HERO
BULL
Aldebaran
Jupiter
S
E
W
MAIDEN
Mars
LION
Moon
Procyon
Betelgeuse
pica
Rigel
CROW
Sirius
RIVER
GREAT DOG
Canopus
KEEL
S
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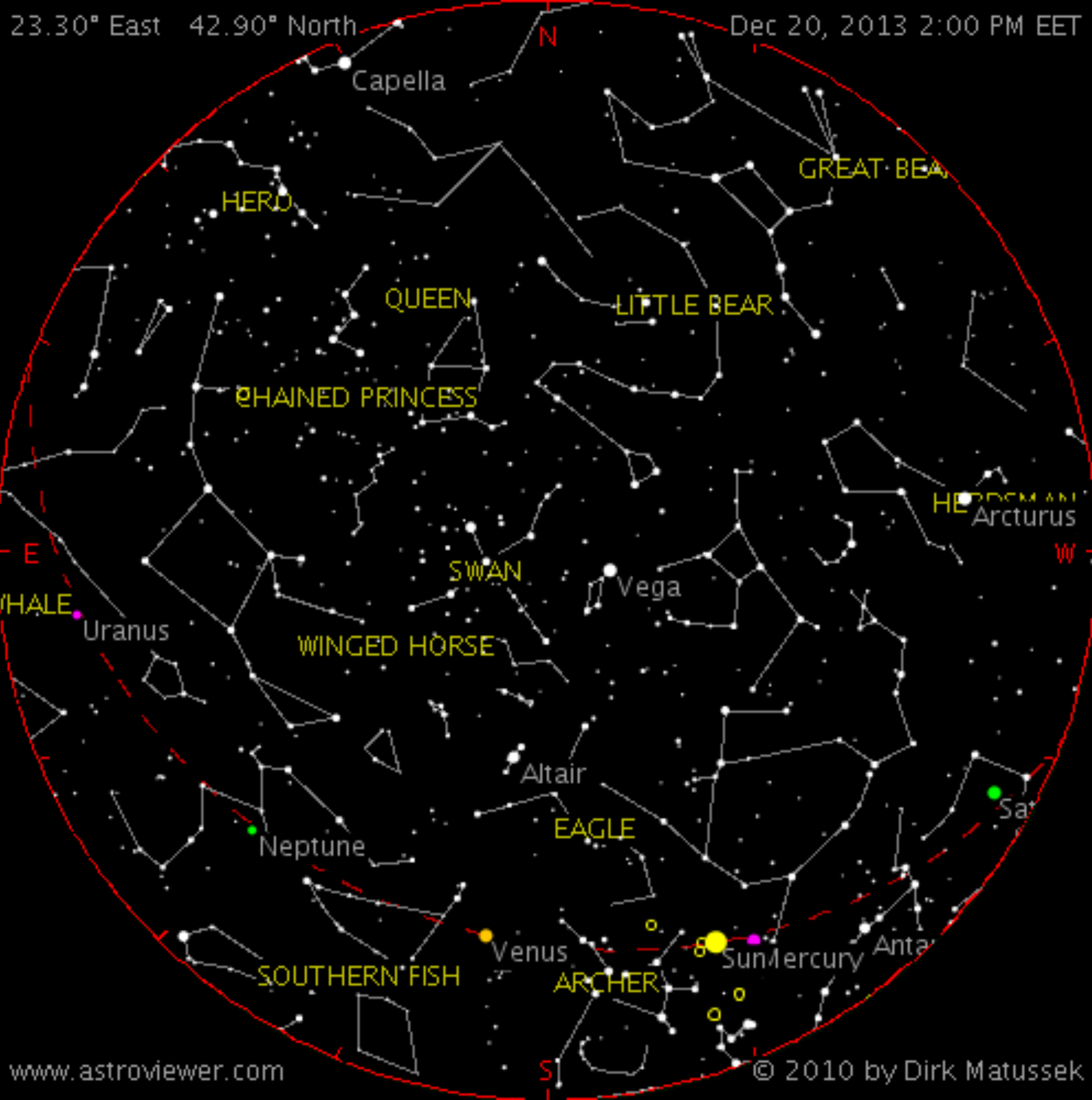
23.30° East 42.90° North
Dec 20, 2013 2:00 PM EET
N
Capella
GREAT BEA
HERO
QUEEN
LITTLE BEAR
CHAINED PRINCESS
HERDSMAN
Arcturus
E
W
SWAN
Vega
HALE
Uranus
WINGED HORSE
Altair
EAGLE
Neptune
Sat
Venus
Sun
Mercury
Anta
SOUTHERN FISH
ARCHER
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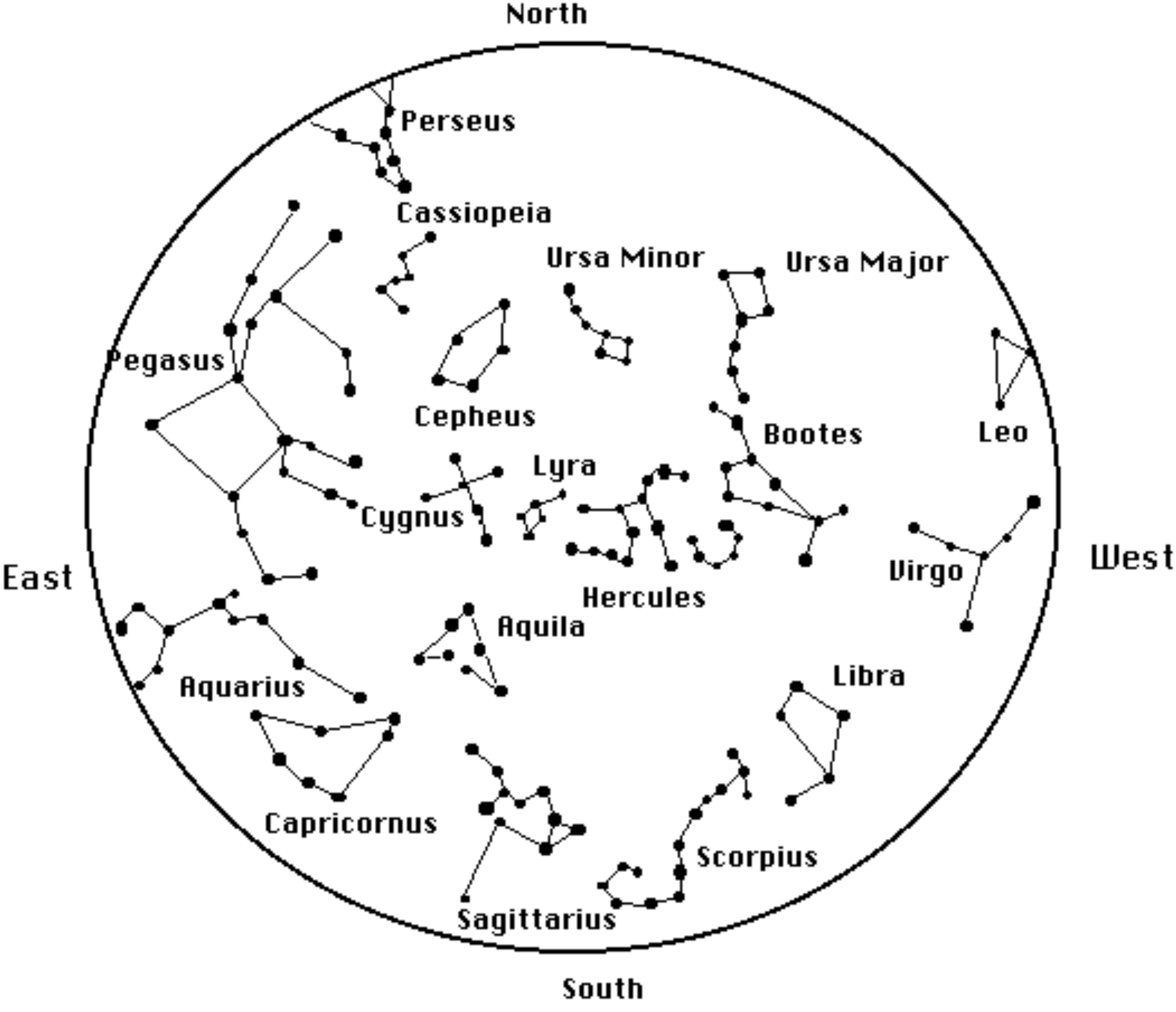
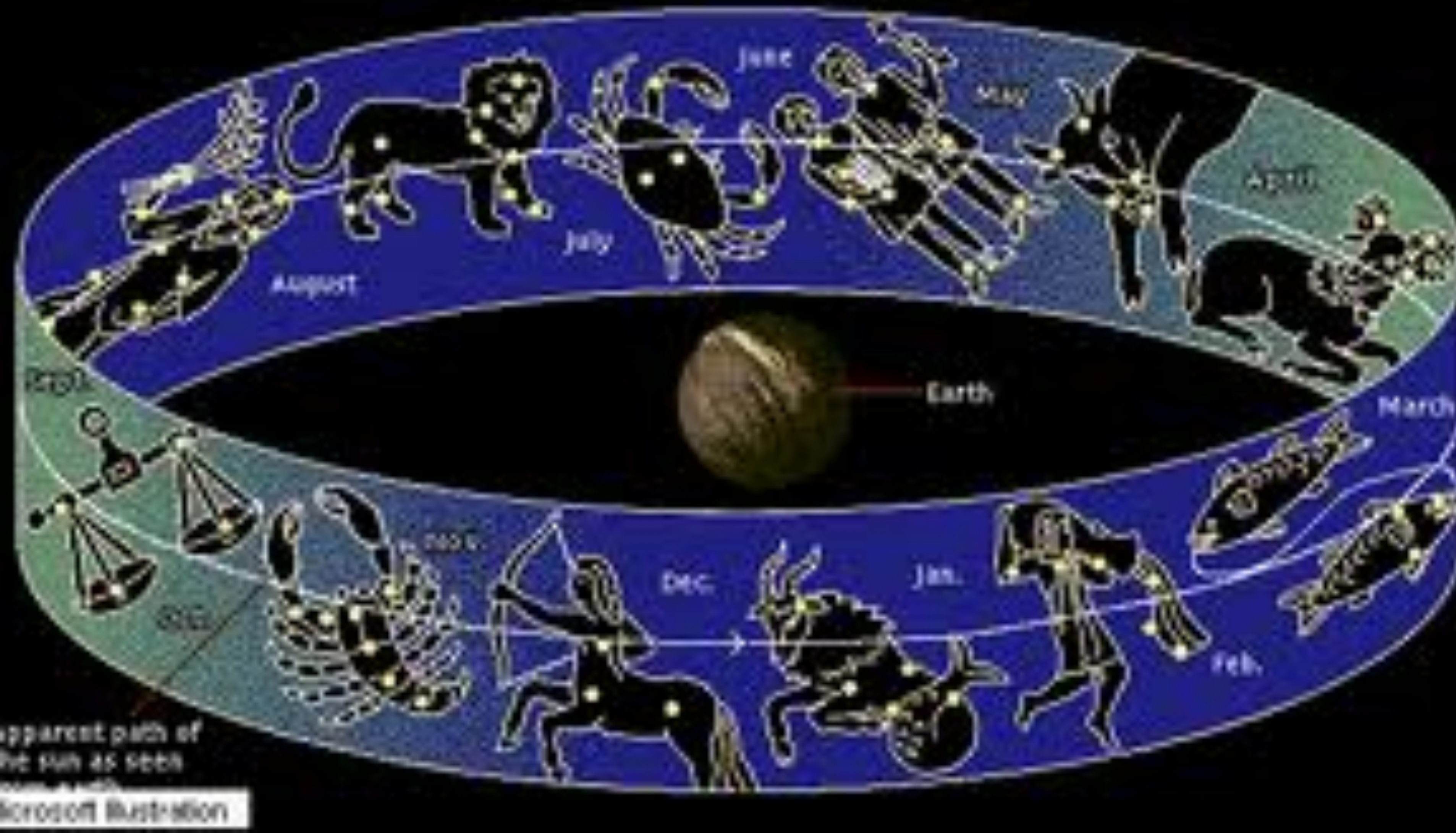
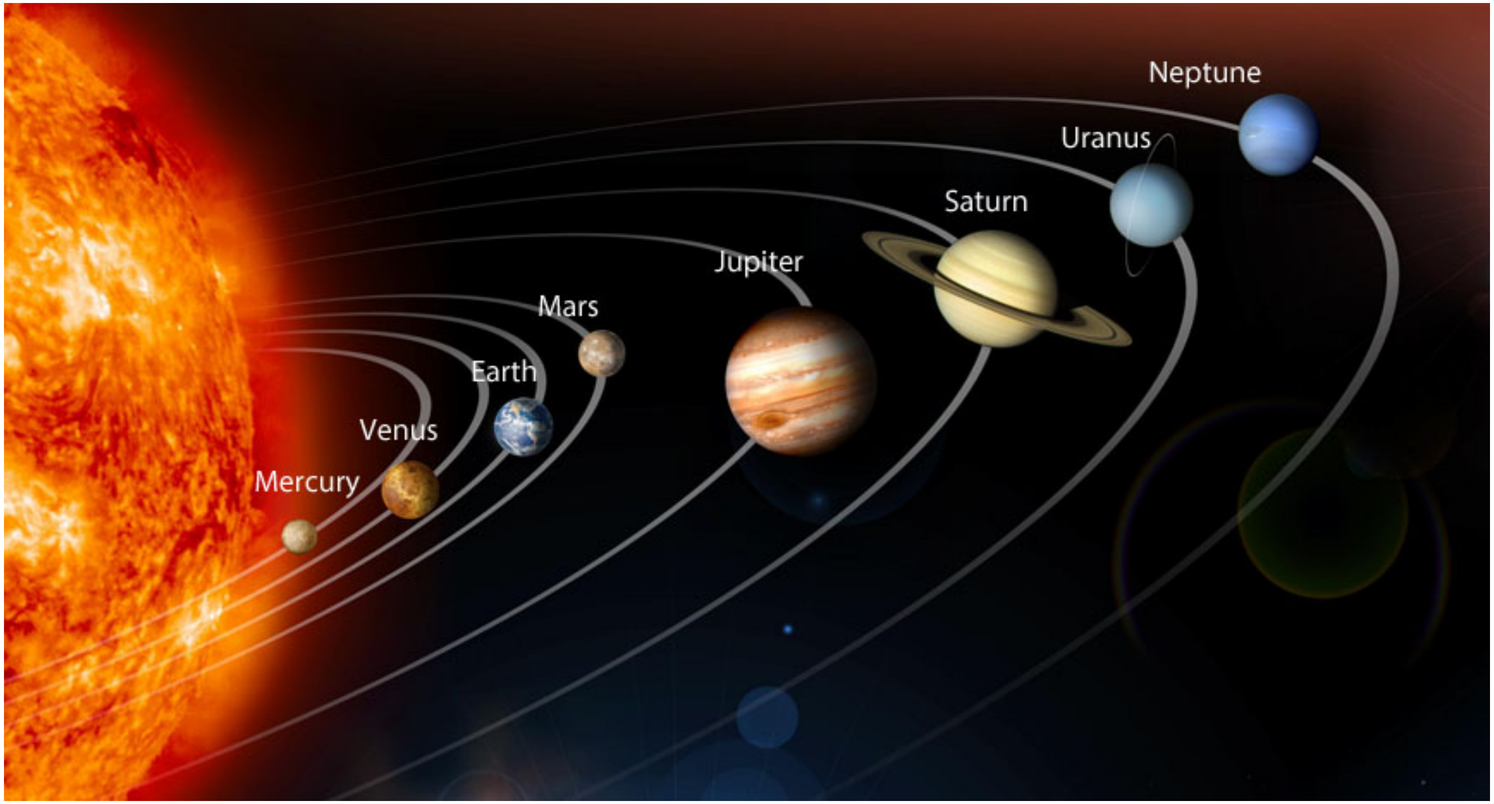

North
Perseus
Cassiopeia
Ursa Minor
Ursa Major
Pegasus
Cepheus
Bootes
Leo
Lyra
Cygnus
East
West
Virgo
Hercules
Aquila
Libra
Aquarius
Capricornus
Scorpius
Sagittarius
South


paws
LEO MINOR
(LITTLE LION)
U.F.O. Galaxy
NGC2683
Castor
GEMINI
(THE TWINS)
Pinwheel
Cluster
M37
Pollux
MA BERENICES
ERENICE'S HAIR)
head
M35
Algieba
Eskimo
Nebula NGC2392
M44
Beehive
Cluster
LEO
(LION)
tail
CANCER
(THE CRAB)
club
Denebola
M65
M66
M105
M96
M95
Regulus
M67
CANIS MINOR
(LITTLE DOG)
Hubble's Variable
NGC2261
Betelgeuse
horn
Rosette Nebula
NGC2237
Procyon
head


Orion


June
May
April
July
August
Earth
March
Dec.
Jan.
Feb.
pparent path of
he sun as seen
icrosoft Illustration

410-350 BC **Eudoxus of Cnidos** – describes the motion of the sun, the moon and 5 planets as rotations of 7 solid shells.
This model persisted for 2000 years.
The 5 planets known then:
Mercury, Venus, Mars, Jupiter and Saturn
The Greek names:
Hermes, Aphrodite, Eras, Zeus, Cronus
They failed to observe Uranus and Neptune,
since despite their huge sizes they were far and therefor faint.



Planet diameters are to scale, but Orbit radii are not.

_**Aristotle 384-322 BC** - divides the world beyond the moon made of perfect crystal spheres, and the world beneath the moon, with continuous variations with four elements – earth, water, air and fire.

Aristarchus & Hipparchus 2-3 century BC - propose that the sun is at the center of the universe (heliocentric system). Hipparchus uses epicycle (secondary circles rotating on a primary circle) in Eudoxus geocentric model 150 years before him.

Julius Ceasr & Sosigenes 45 BC - Sun year with leap year every 4 years, based on 365.25 days per year.

Hero of Alexandria 60 BC – writes the book "Metrics, Mechanics & Pneumatics". Built a steam engine. Measure (mistakenly) the earth diameter.

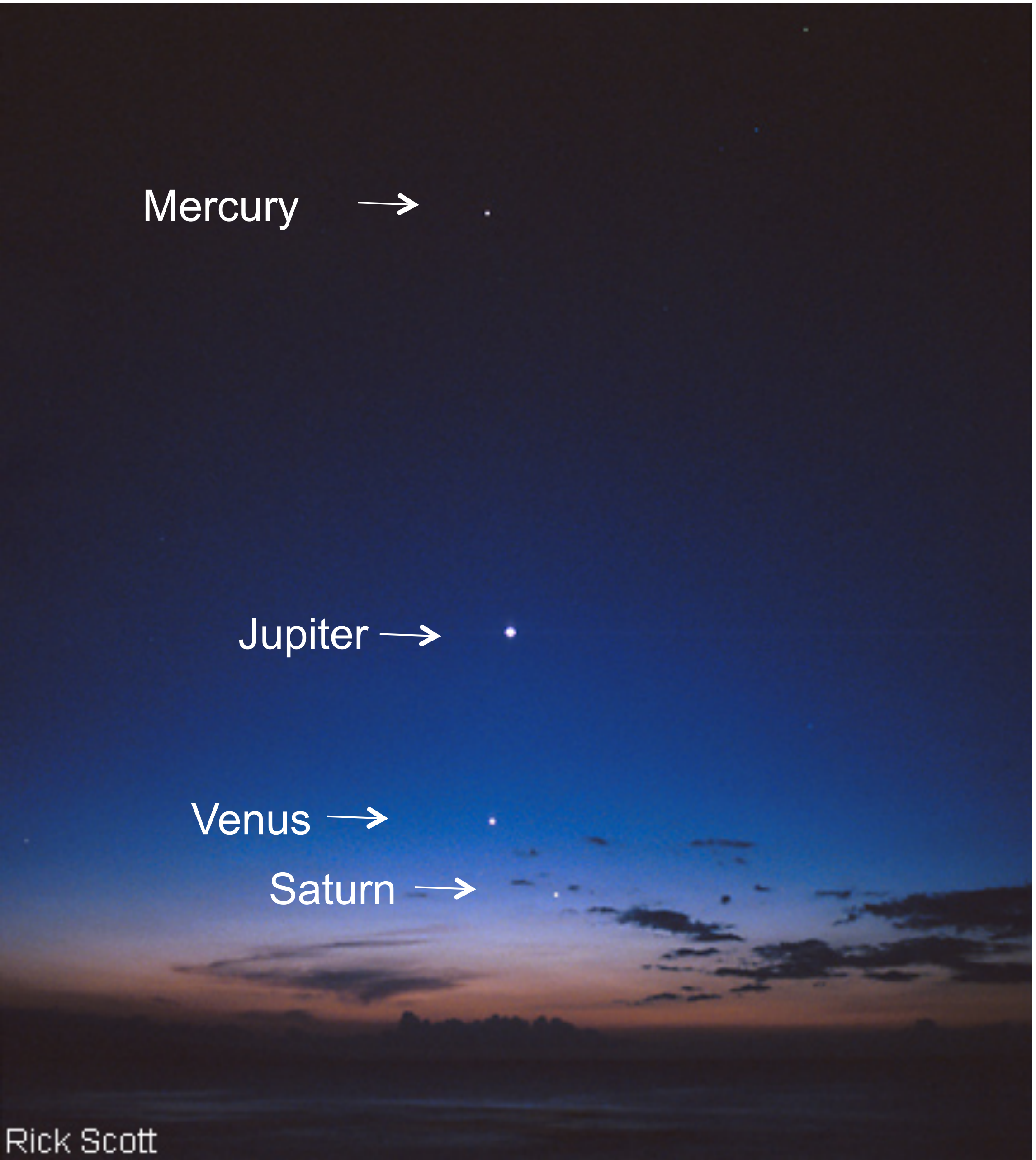
Seleucus 150 BC - related tides to the position of the moon.

Claudius Ptolemy 130 AC – established a cosmological model that persisted 500 years: Star move at constant speeds in epicycles and equants that grow in size from the moon, Mercury, Venus, Jupiter and Saturn, the earth is at the center. Beyond are the stable stars with circular motion caused by the "nine inner layers of heaven".

The persistence of the geocentric model (and rejection of the heliocentric model) was mainly due to religious belief (Man is the center of creation), and maybe also by the difficulty of describing mathematically motion measurements from non-inertial systems. Measuring efforts (till early Renaissance) were not sufficiently precise to support one model or the other. It is amazing though that epi-circles inherently describe that what we see is rotation of a planet in a circle around another circle, not ourselves. Yet it drove only a few to suggest that the sun and not earth, is the center of their rotation.

Ptolemy’s model of earth (center) and the stars

http://www.opencourse.info/astronomy/introduction/

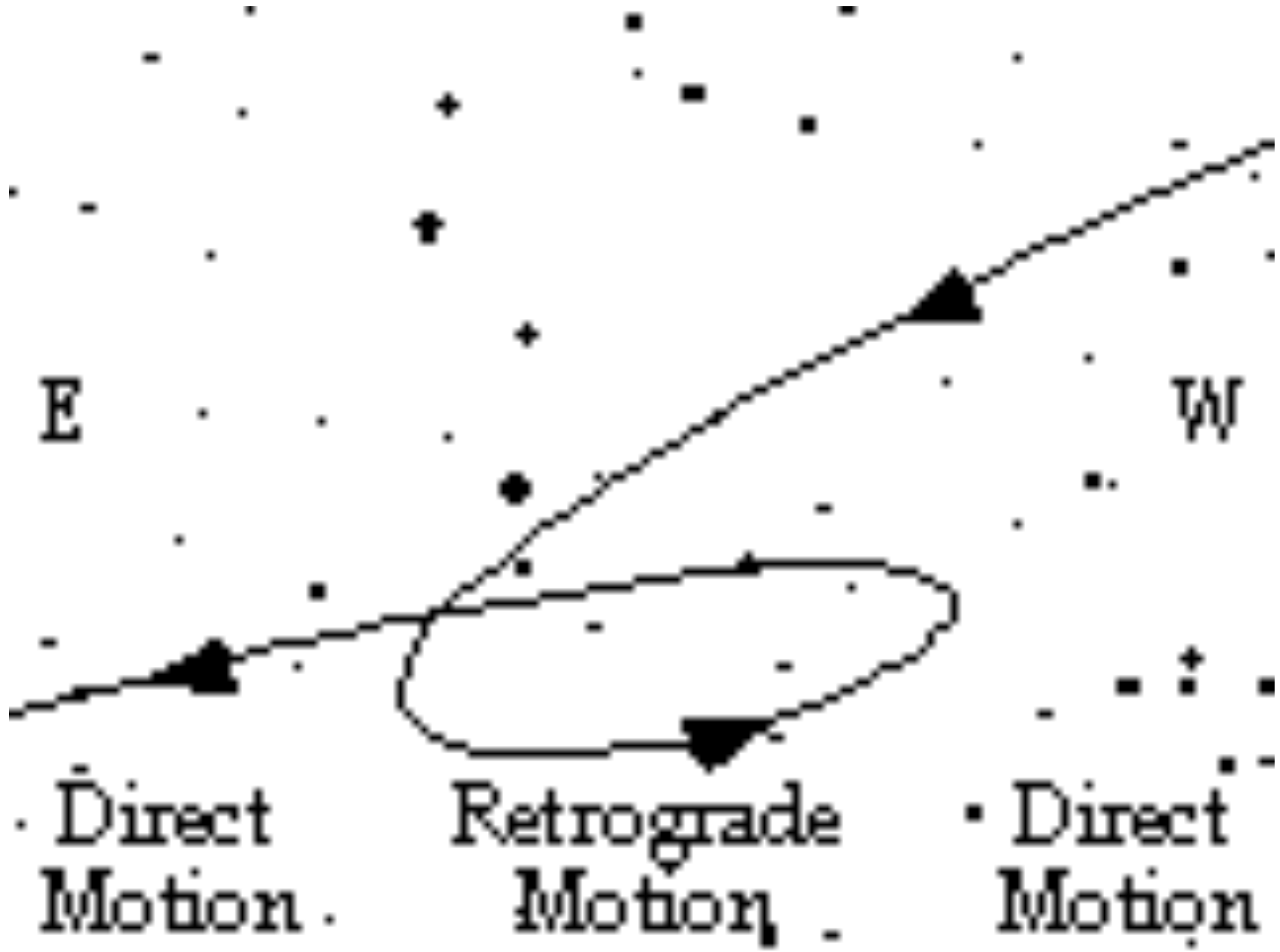


The sun, the moon and the planets all orbit in planes close to the Eclipse.
This beautiful picture display 4 planets in one line with the sun right after sundown.

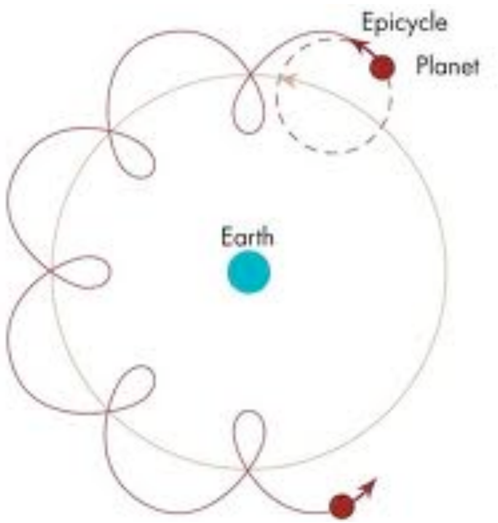
Since Mercury and Venus orbit radii are smaller than ours, they are always near the sun, and can be seen at dark close to dawn or sunrise only.
Other planets with radii larger than ours move all over the sky.

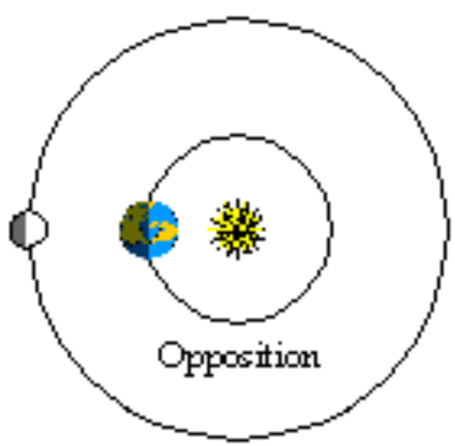
The planets, wonderers in Greek, move relative to the permanent stars. In the following movie Jupiter (the brighter) and Saturn migrate over the aligned star background, displaying backwards motion. This was puzzling for a long time.

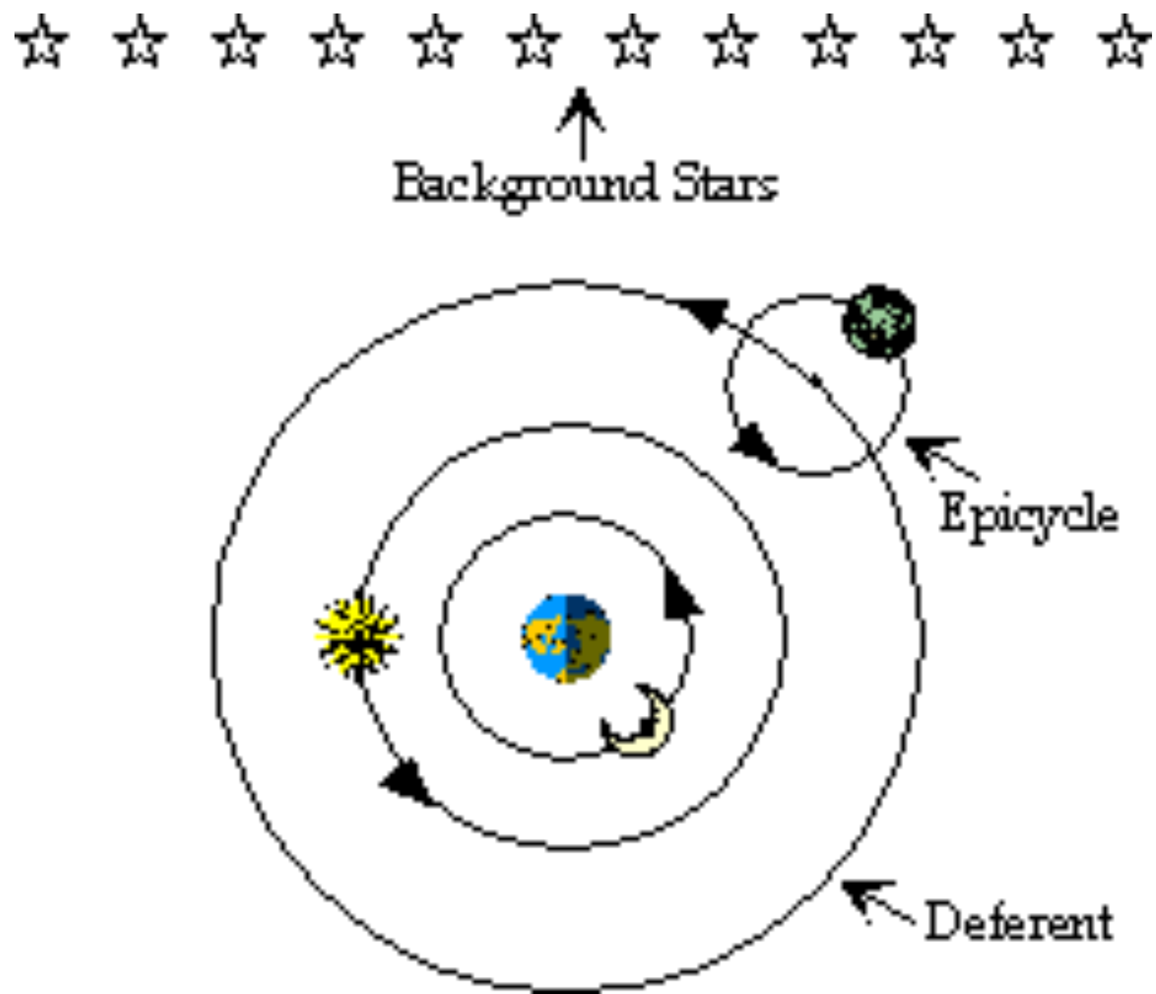
We understand the seemingly backwards motion of a slow car looked at from a bypassing fast car. The effects are complex for circular orbits.
Therefore, the relative motion of planets detected from earth slowed down and accelerated, and was wrongly linked to distance from earth.



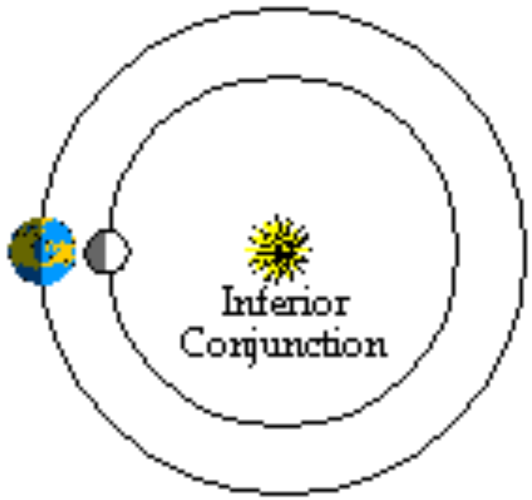
Plato - explained the backward motion by epicyclical orbits around earth.
This model failed to explain variations in brightness (specially evident for Mars).
But star brightness could not be measured in antiquity. Even relative brightness to stable stars was a subjective evaluation with low precision, certainly when it had to be repeated over after very long times.

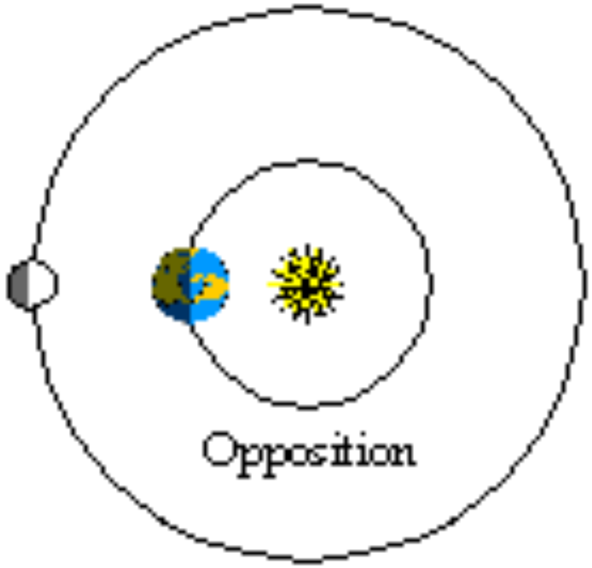






Measurements of the periods for planet orbits must be corrected for earth motion. This is specially significant for Mars, Venus and Mercury since their orbit radii is close to that of earth.





384–322 BC **Aristotle (Aristo)**

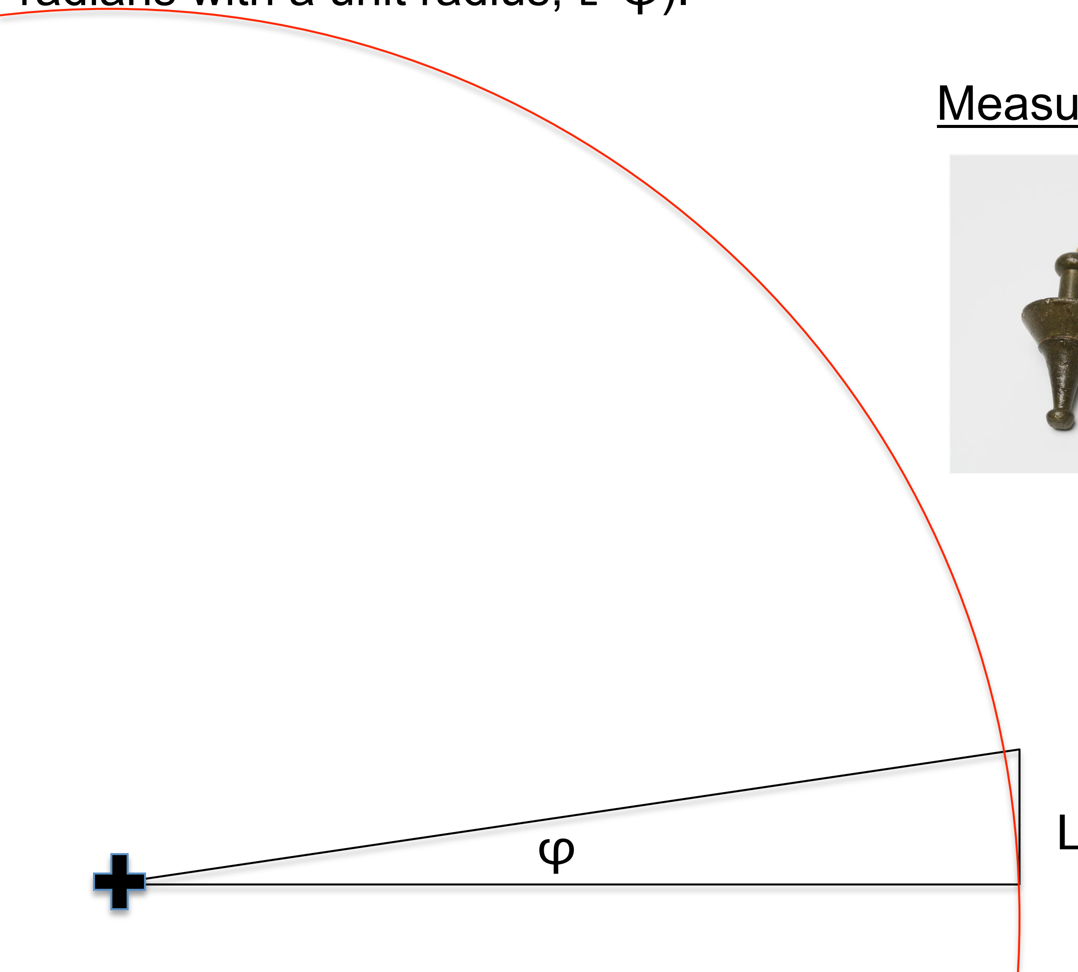
Aristo – listed proofs that earth is a sphere:

1. A sphere is a perfect shape.
2. The horizon is circular.
3. A ship sailing away to open seש “sinks” into the horizon.
4. The earth shadow on the moon is circular.
5. Sailing south the southern stars “rise” upwards above the horizon.
6. Objects fall everywhere towards the center of earth.

The god Atlas carries the spherical crystal of the stars.
(often mistaken as the Globe of earth)

Measurements of star positions is based on their angles with other stars.
Angles and distances are related by proportions of similar geometrical shapes (e.g. of triangles). Since distances to stars were hard to establish (successfully measured to the moon and the sun only by the Alexandria school), star maps were interpreted as if all stars are on one (or several) solid spheres.

For small angles the arc angle and the tangent are almost equal, (numerical equal for radians with a unit radius, $L=\varphi$).

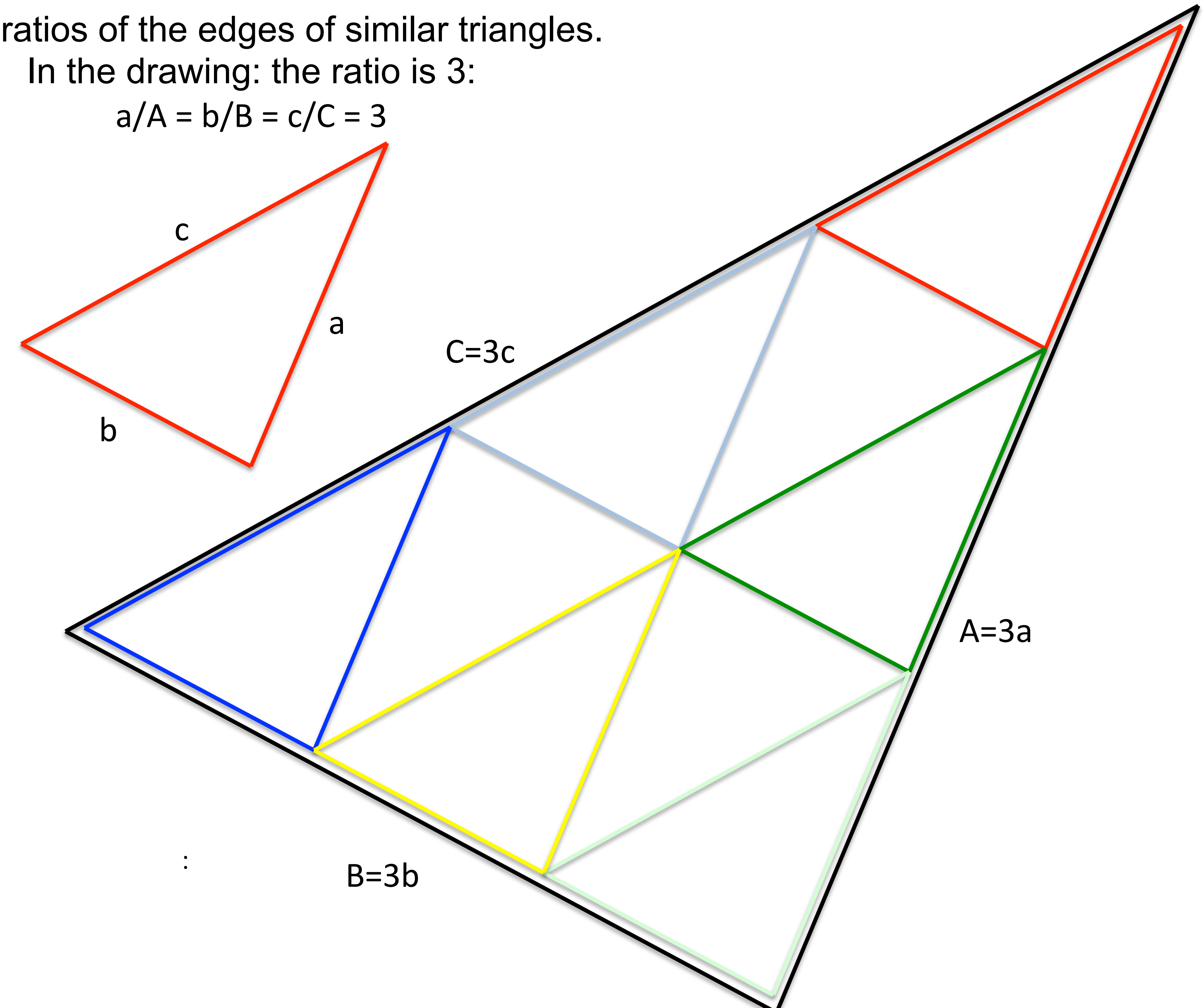


<u>Measuring angle with respect to the horizon</u> – plumb-line, plummet.

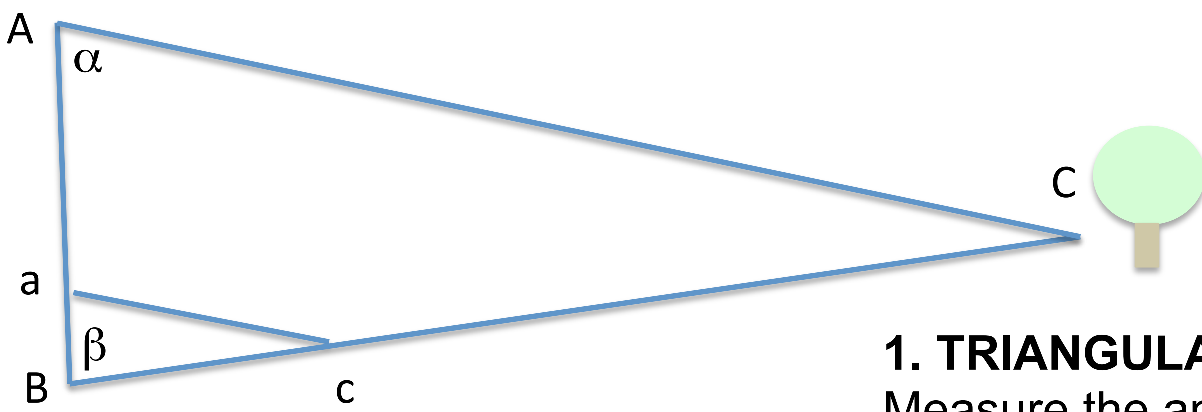
<u>Measuring angles between two stars</u> – Sextant

Equal ratios of the edges of similar triangles.
In the drawing: the ratio is 3:

a/A = b/B = c/C = 3



How can we measure a distance to a far object without getting to it:



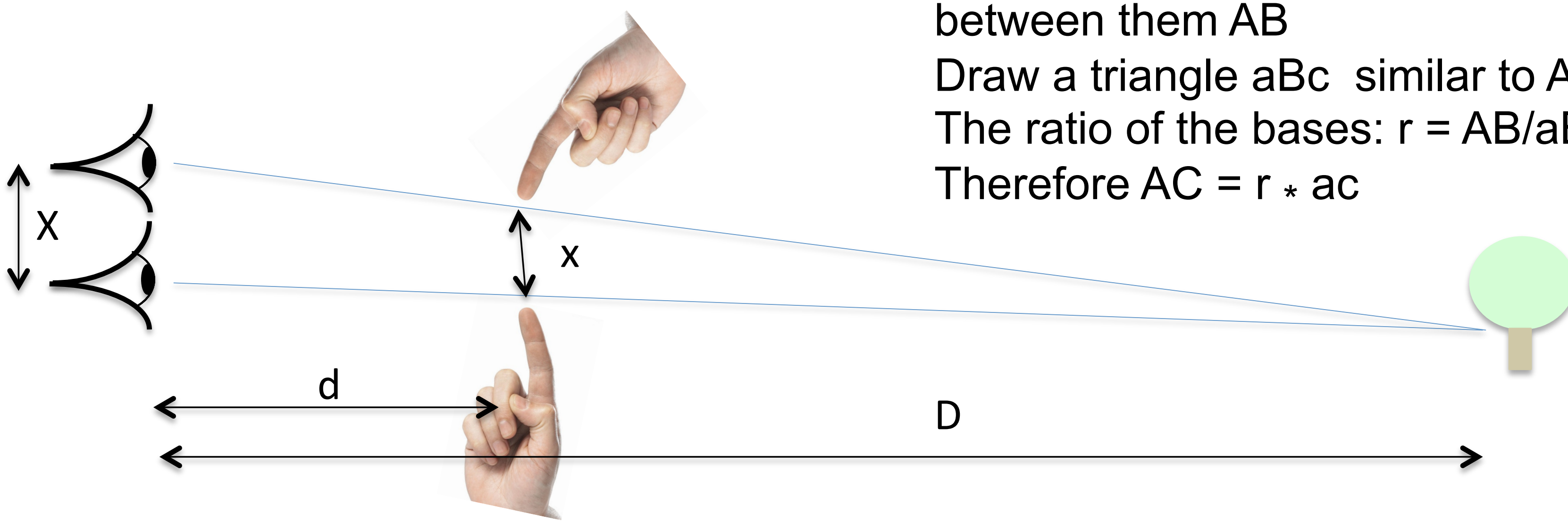
1. TRIANGULATION:

Measure the angles α,β from two positions, A & B, and the distance between them AB

Draw a triangle aBc similar to ABC

The ratio of the bases: r = AB/aB

Therefore AC = r * ac

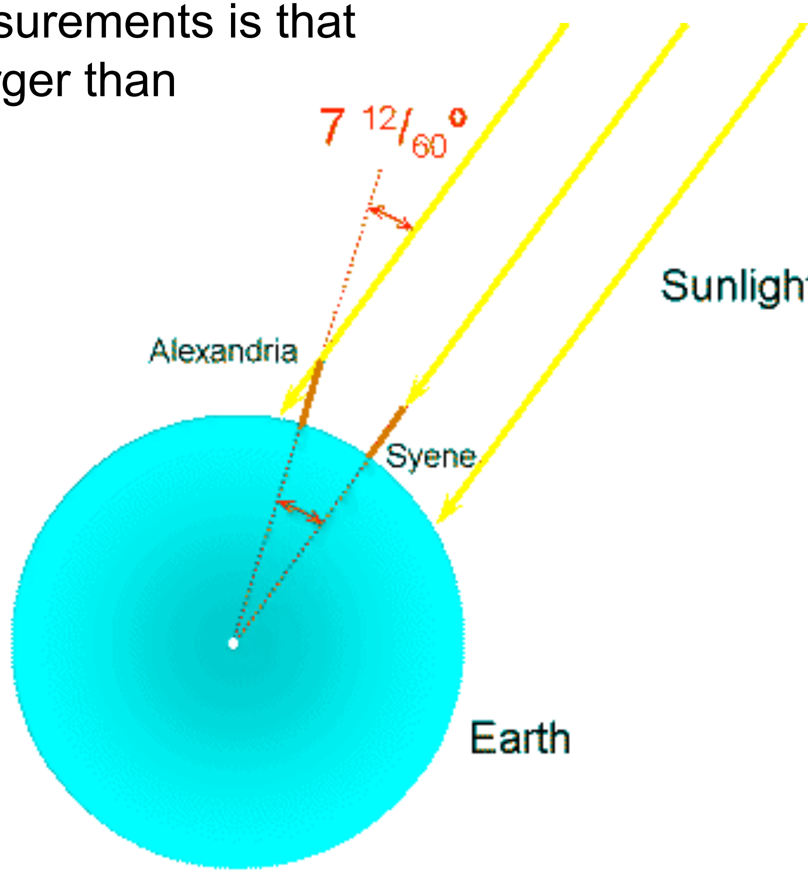


2. PARALAX:

Hand out a finger. The finger is at distance d from the eyes, and the distance between the eyes is X. If the finger need to move a distance x sidewise to points at a far object seen by each eye at a time, its distance is D = d * X/x

Eratosthenes 250 BC Measured the earth diameter:
When the sun is exactly up in the sky at Syene, (a vertical stick has no shadow), the sun angle at Alexandria was $7^o12' = 7.2^o$.
From the measured distance between Syene and Alexandria, 925 km, the earth diameter is $925 \times 360 / 7.2 = 46,250$ km

How did Eratosthenes know to perform the measurement at the same time at two distant places? He anticipated sun eclipse, and prepared two measurements of sun angles at the two sites at the moment that the moon just touched the sun perimeter. Implied in his measurements is that both the sun and the moon distances are much larger than the earth radius.

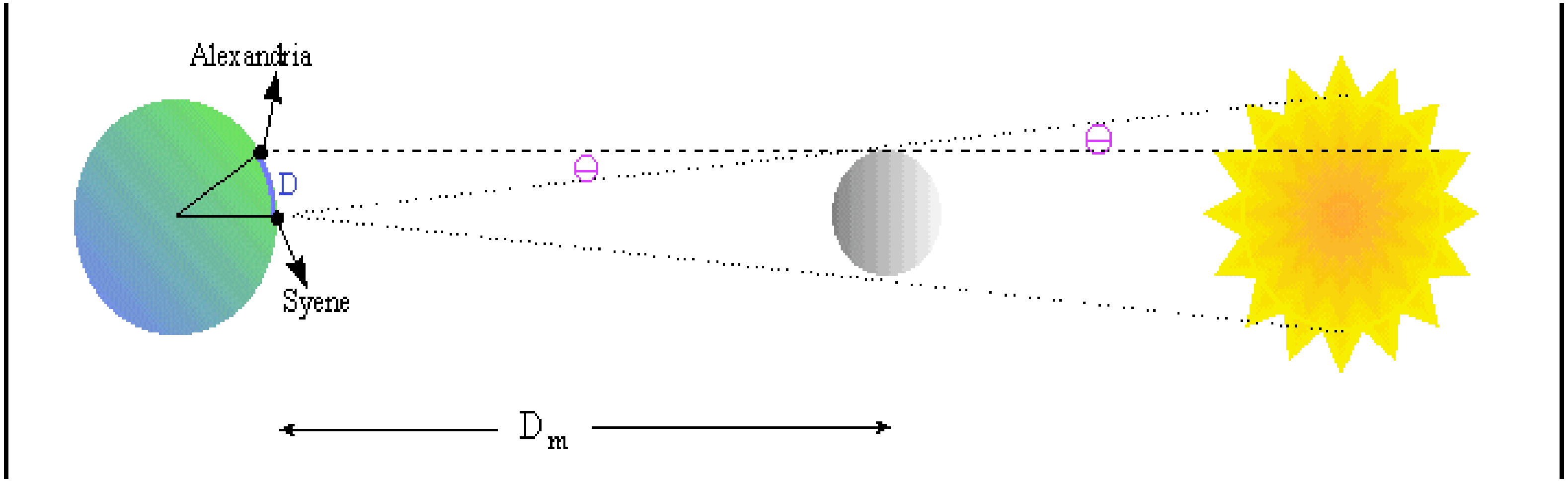


The present value of earth diameter is 40,070 km (Radius of 6,400 km).

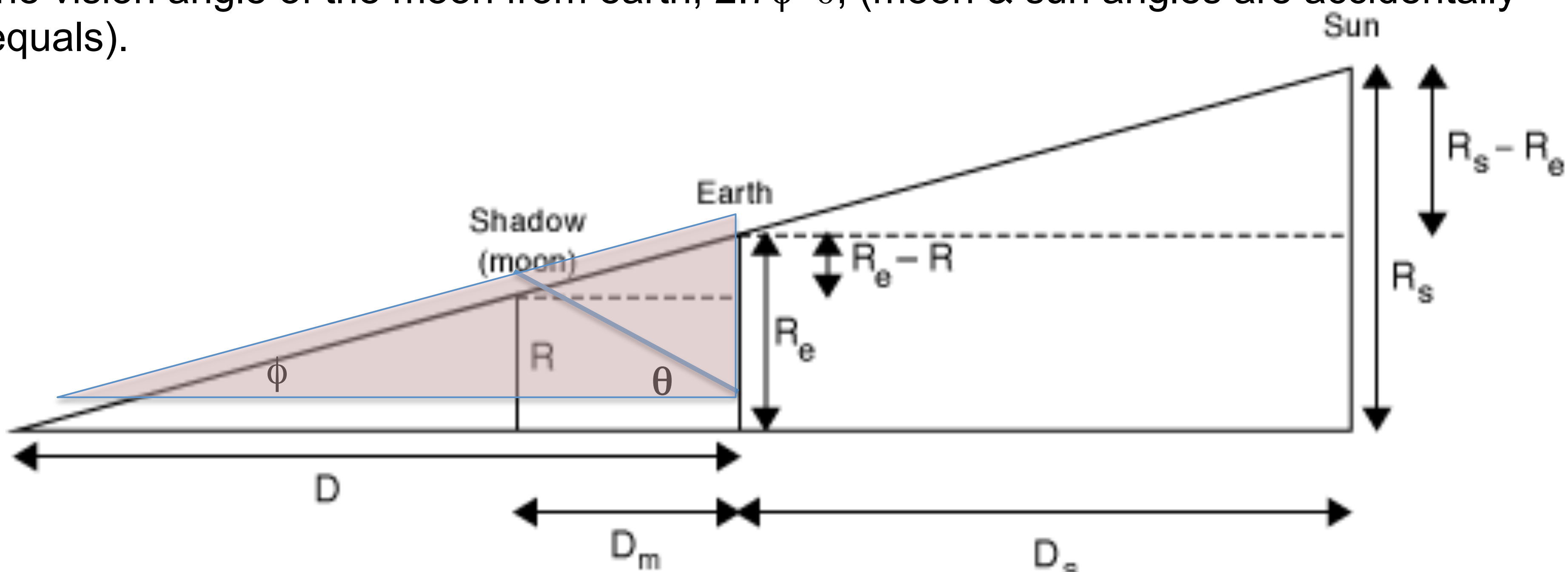
How would you increase the precision of angle measurements?

190-120 BC Hipparchus of Nicaea

Prepares the first catalog of stars, located by latitude and longitude angles.
Denied the heliocentric model.
Measured distance to the moon by parallax:
The viewing angles of the sun and the moon = 0.5^o (exactly 0^o 33' 14").
During complete sun eclipse, the moon exactly covers the sun.
When this happens in Syene, 1/5 of the sun is seen covered in Alexandria.
corresponding to an angle of θ = 0.1^o = 0.001745 Radians. (1^o = π/360 radians)
Therefore the distance to the moon = 925 / 0.001745 = 530,000 km
In fact, since at the time of the eclipse the sun angle φ was not 90^o but $\cos\varphi \sim 0.7$
this factor should be corrected, giving moon distance of 371,000 km
The present value is 384,403 km.



The moon diameter from the time from start till end of a moon eclipse by earth:
Earth shadow is passed by the moon in 10.8min, (corresponding to an angle 2.7 times the vision angle of the moon from earth, $2.7\phi=\theta$, (moon & sun angles are accidentally equals).



The vision angle of the moon $2\phi = 0.5^o = 0.0087$ Radians,
The radius of earth $R_e = 6,400$ km
The length of the shadow cast by earth **$D = R_e/\phi = 1,472,000$ km**
The angle of vision of the earth shadow $\theta = 2.7\phi$
Therefore **$R=\theta D_m$ $R=\phi (D-D_m)$ => $D_m(\theta+\phi)=\phi D$**
Thus the distance to the moon and the moon radius: (today radius=1,737 km)

$$D_m = \phi D / (\theta+\phi) = D/(1+\theta/\phi)=D/3.7=398,000 \text{ km}$$

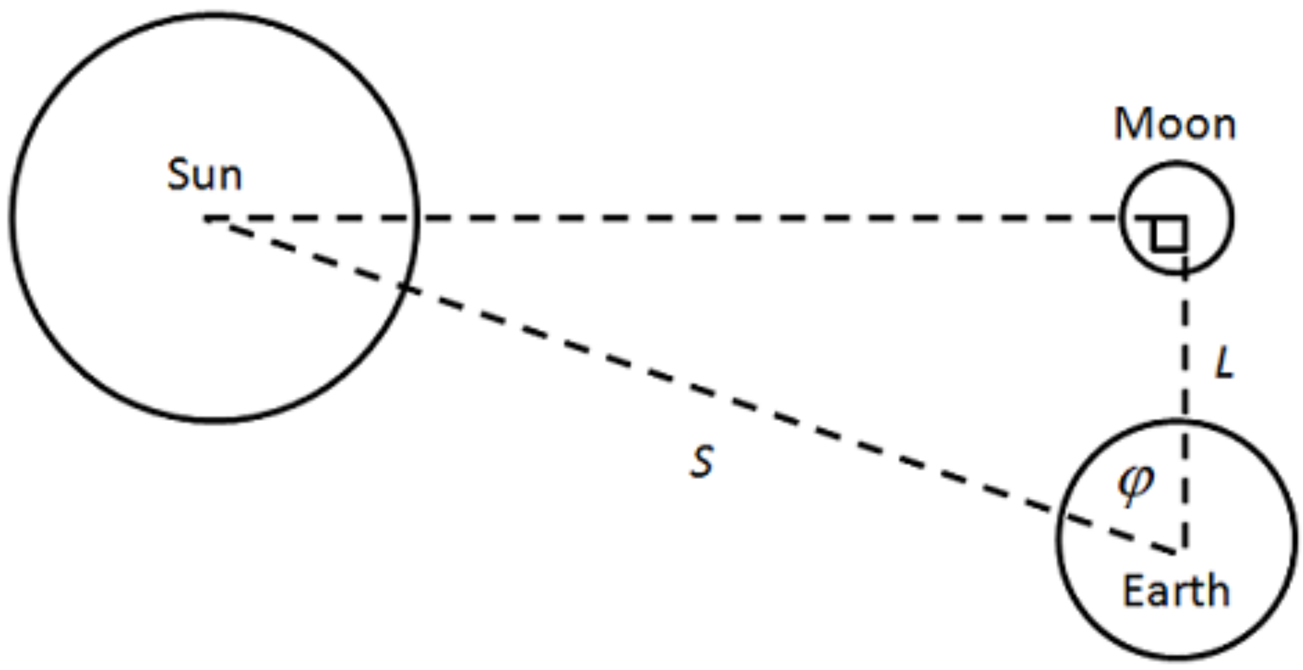
$$R_m = \phi D_m = 0.0087/2*398,000 = 1,731 \text{ km}$$

310-230 BC Aristarchus of Samos

Believes in the heliocentric model.
Measured the distance to the sun, based on the distance to the moon:
When the moon looks exactly half-circle, the sun-moon axis is perpendicular to the moon-earth axis. He measured the angle between the sun and the moon, $\varphi = 87^o$, and concluded that $S/L = 1 / \cos\varphi = 19$. The error is large (why?).

The present value is: 149,600,000 / 384,000 ~ 390 corresponding to ~ 89.9º

"Jumping" from one known distance to a much longer distance is a common method in astrophysics till today.



The sky as a time clock

Sun – days. Sun height angle at noon – the season in the year
Moon – months

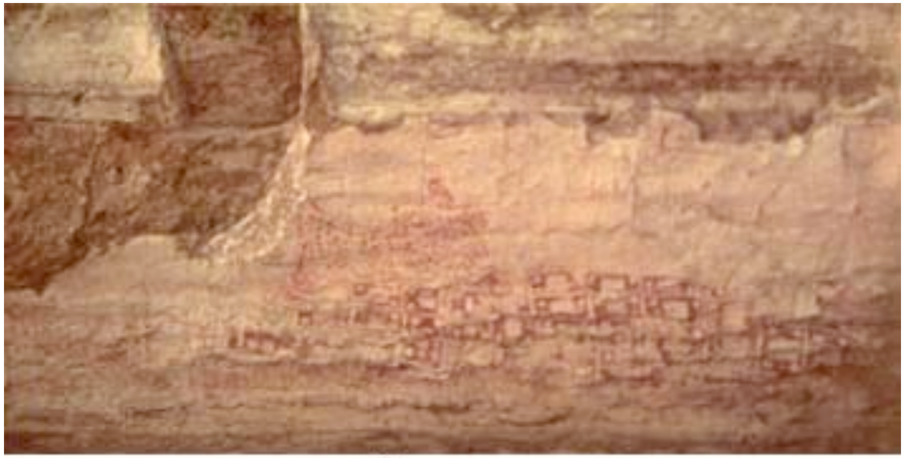
Accumulated listings of star measurements indicated that their sky locations is not reproduced exactly every 12 moon months. Also a moon month is not an integer number of days. Astronomers realized that the ratios between earth rotation, sun and the moon cycles are not a simple rational number, and determined "corrections" every few years. As the accuracy of measurements increased, these corrections were set in advance for many years.

In China- 1281 AC 12 years cycles: (tiger, rabbit, dragon, snake, horse, goat, monkey, rooster, dog, pig, rat, ox) – to retrieve the same star locations.
Records of star locations started at the 6^{th} century BC.
Zhang Heng 78-139 AC cataloged 2500 stars.
13^{th} century – 27 observatories in China. Year – 365.2325 days.
In India – 1200 BC Vedanga Jyotisa - measurements of star migration for religious purpose. Alexander the Great brought Hellenistic astronomy to India.
476-550 AC Aryabhata – decimal numbers system. Applied trigonometric tables to measure star motion, eclipses and length of the year.
598-668 Brahmagupta – the mathematician, developed a method to calculate star locations. Claimed that earth is a sphere and moves in the universe. His writings were translated at 770 AC to Arabic at the court of **Caliph Manzur**.
The Maya – kept separated agricultural and religious calendars.

MAPS

Ancient Babylonian maps (right)

The map actually draws the city as seen e.g. from a nearby mountain, and depict houses, river and the city walls.



Egyptian maps (left) – draw the borders of agricultural lots, to be recovered after Nile overflows

Homer – the earth is flat and round, surrounded by oceans.
The Greeks drew maps of the Aegean see islands and bays, based on sailors descriptions.
It is difficult to match visible scenes into larger scale maps. (For example, if you try to draw a circular path through streets in your city, you will not reach back to the same spot).

The solution: location in a coordinate system.
Latitudes and Altitudes can be determined from measurements of star angles.

650 BC Anaximander, a student of Thales,
Drew a world map (lost in time).
Pythagoras understood earth is a sphere

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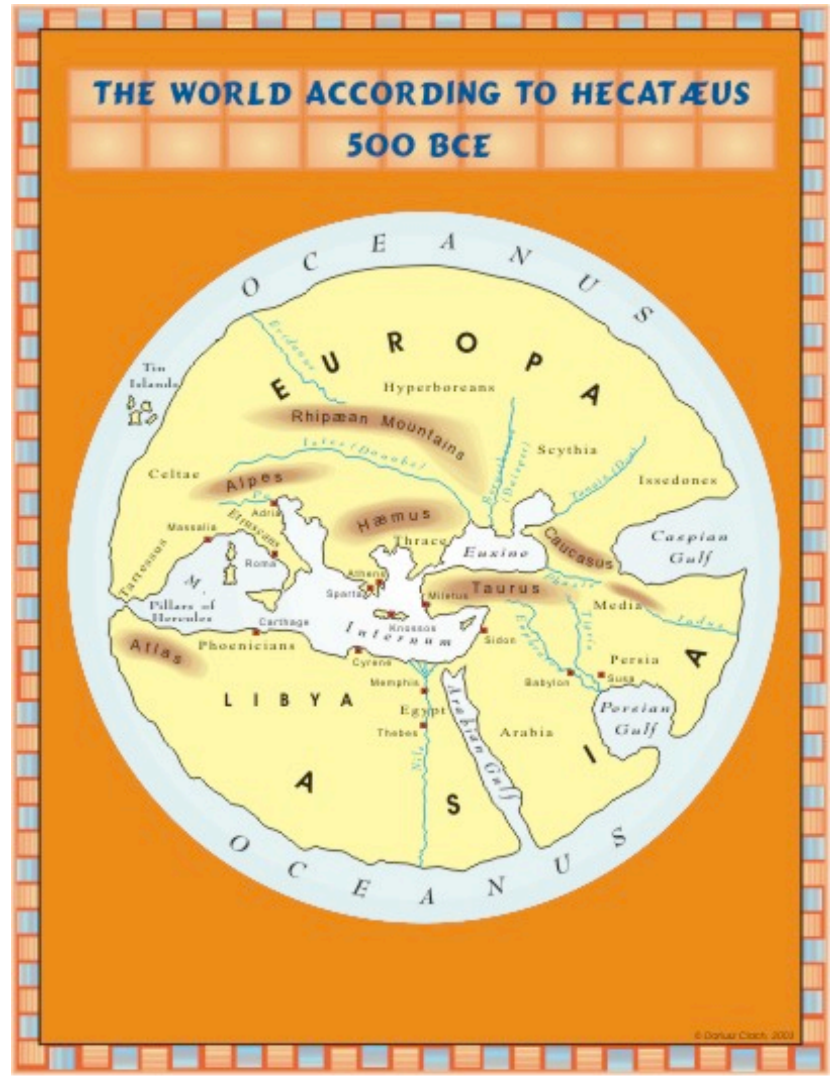
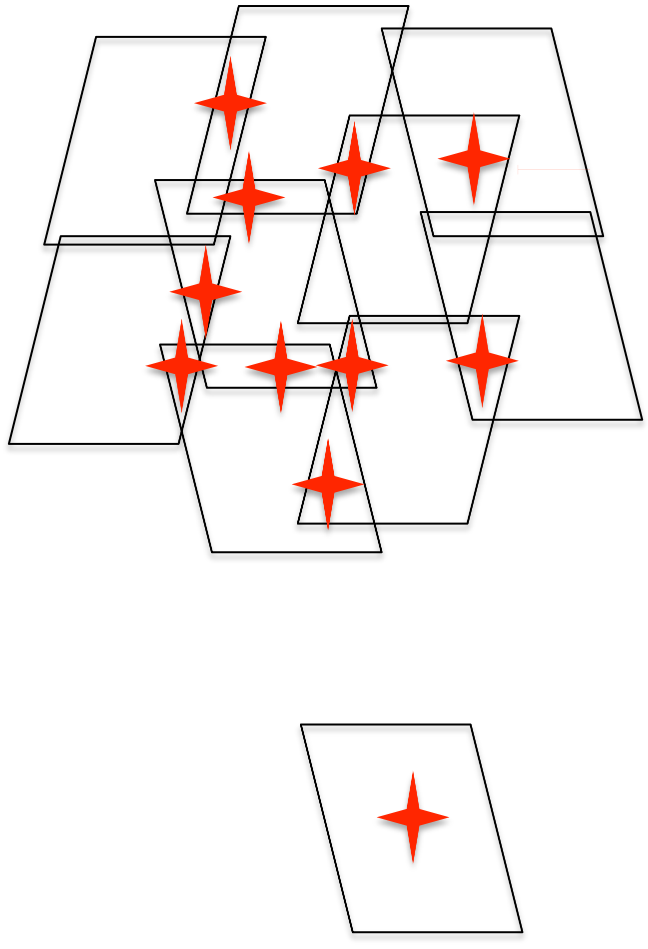


Stitching many local maps into a world map:

1. Match a location (e.g. a city) that appear in two maps. Need orientation.
   Problem: accumulative errors.
   Gaps of unpopulated regions or oceans.

2. Position a location in its known coordinate.



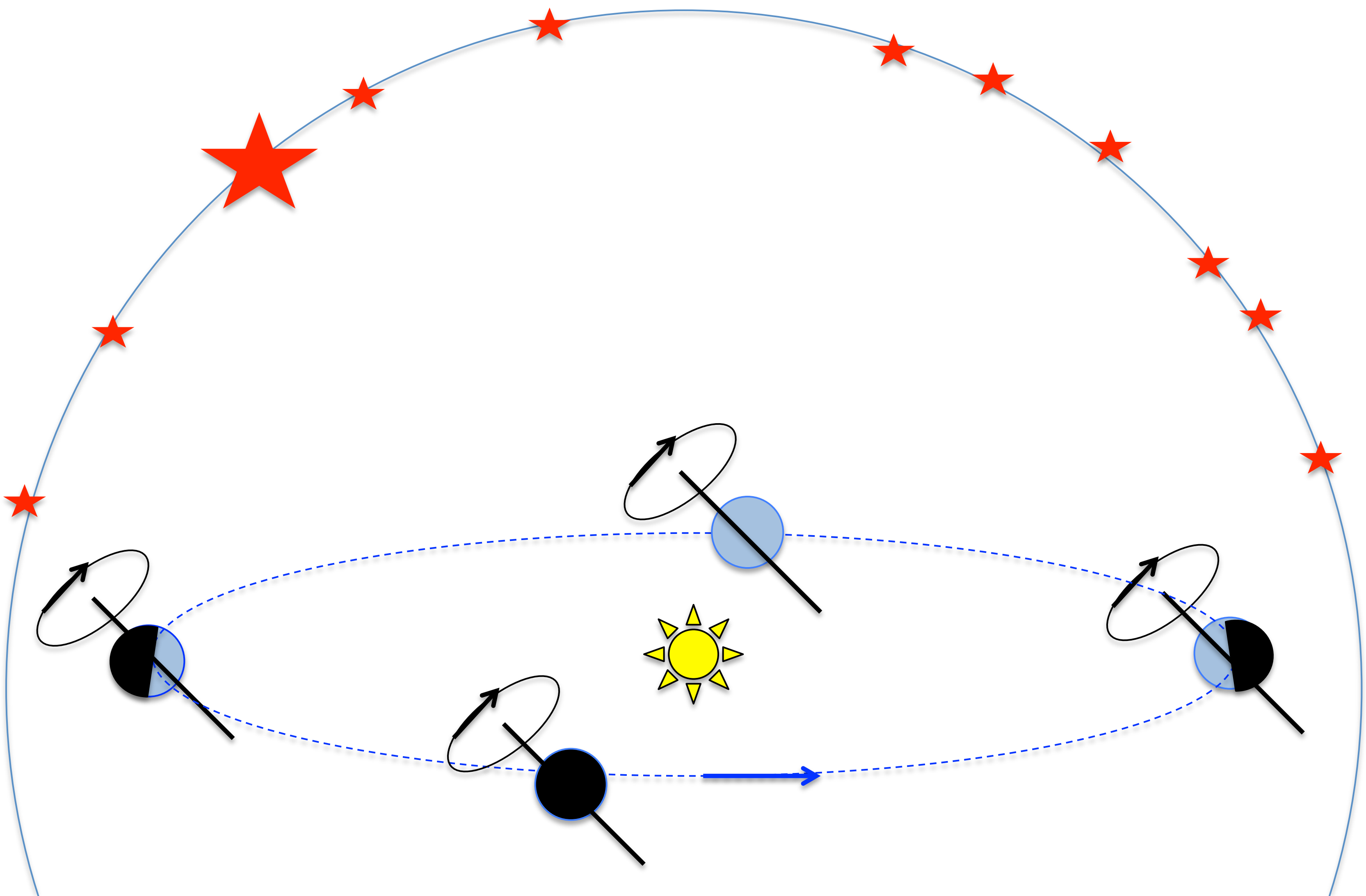
Stars as locators on earth

Maps drawing, Cartography, apply spherical geometry.

Star locations at sunrise or sundown determine longitudinal position on the globe.

The angle of the north star determines the altitude.

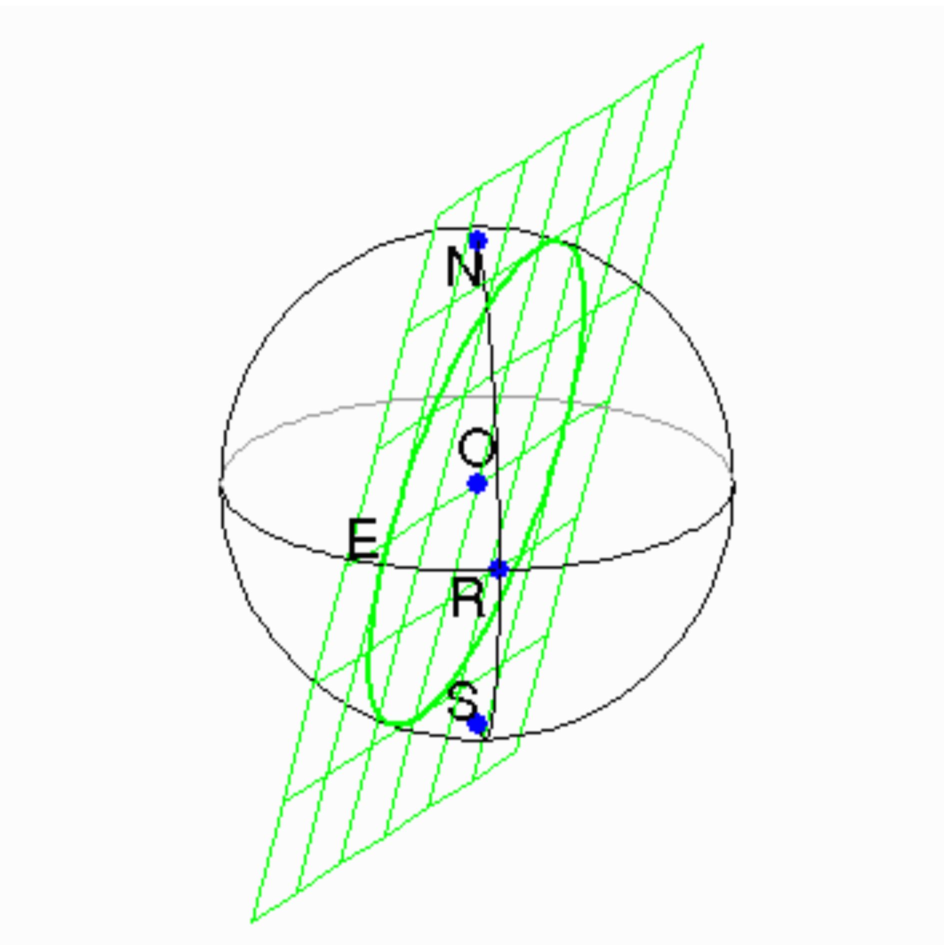
The measurements need correction by the season

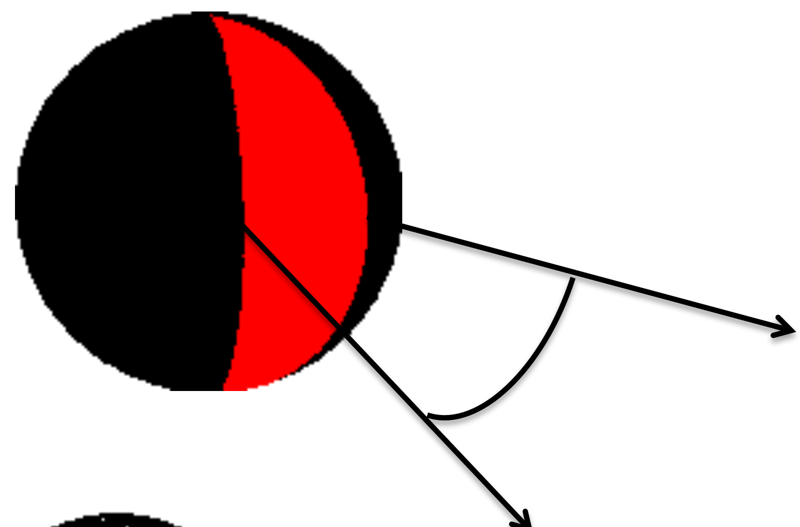


Dicaearchus 300 BC – defined a net of coordinate system
Eratosthenes 250 BC – defined the origin of coordinates by an altitude line from Rhodes to Hercules columns (Gibraltar) (today this line is 36 altitude), and longitude 0 in the Canary islands, (today 18 west longitude).
Hipparchus ca. 190 – ca. 120 BC determined the precession in earth rotation axis as 1 degree in 100 years.
Ptolemy 120 AC – determined city coordinates from star angles.
He drew world maps by two methods of projecting spheres to planes.
He used Sextants and Quadrants to position 1025 stars.
He believed in the geocentric model: earth is at the center of universe.
The 5 planets moved in concentric spirals around earth.
He used spherical geometry for his star maps.

Spherical Geometry:

Large circle on a sphere is a circle in a plan that contains the sphere center.
Spherical angle between two points is the angle between two large circles crossing these points, creating an orange-like slice. (Also the angle between two lines between these points and the sphere center).
A spherical triangle is created by three large circles, crossing each two of the triangle vertices.





Menelaus of Alexandria 1st century AC – describe spherical geometry.
The area of a spherical triangle, Δ, created by three spherical angles A,B & C, in a sphere of radius R (left drawing), is: (later named Girard''s Theorem)

$$\Delta = R^2 (A+B+C - \pi)$$

In a different way: the sum of angles of a spherical triangle:

$$A+B+C = \pi + \Delta/R^2$$

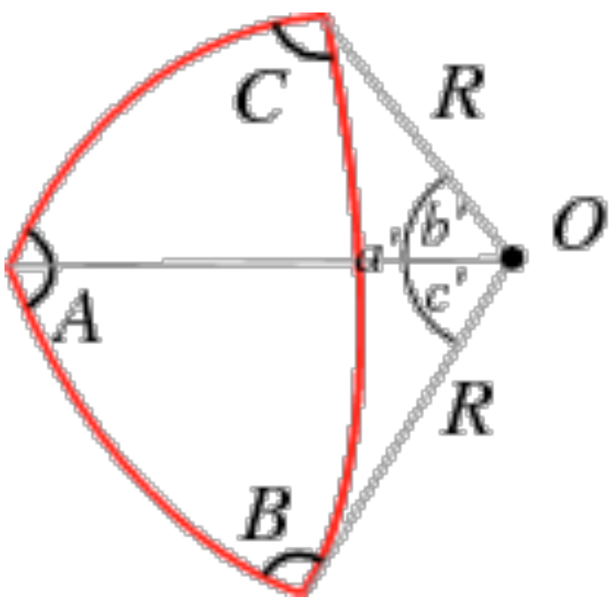
Proof: http://math.rice.edu/~pcmi/sphere/gos4.html

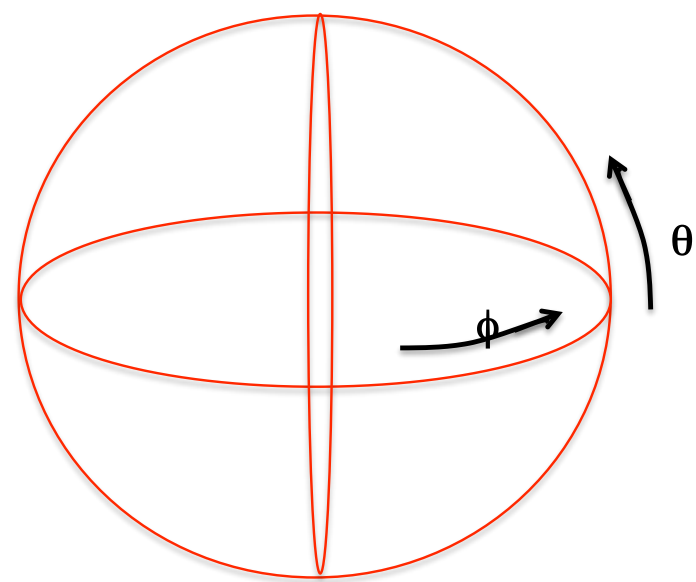
For example (right drawing) , sum of angles of a 1/8th "apple cuts" by three perpendicular planes is 90x3=270°

This is different than planar triangles:

1. Sum of angles depend on area (not π for the planar triangle).
2. spherical triangles with equal angles are identical (not just similar).

The implication to world maps: here is no earth map that preserves both area and angles.





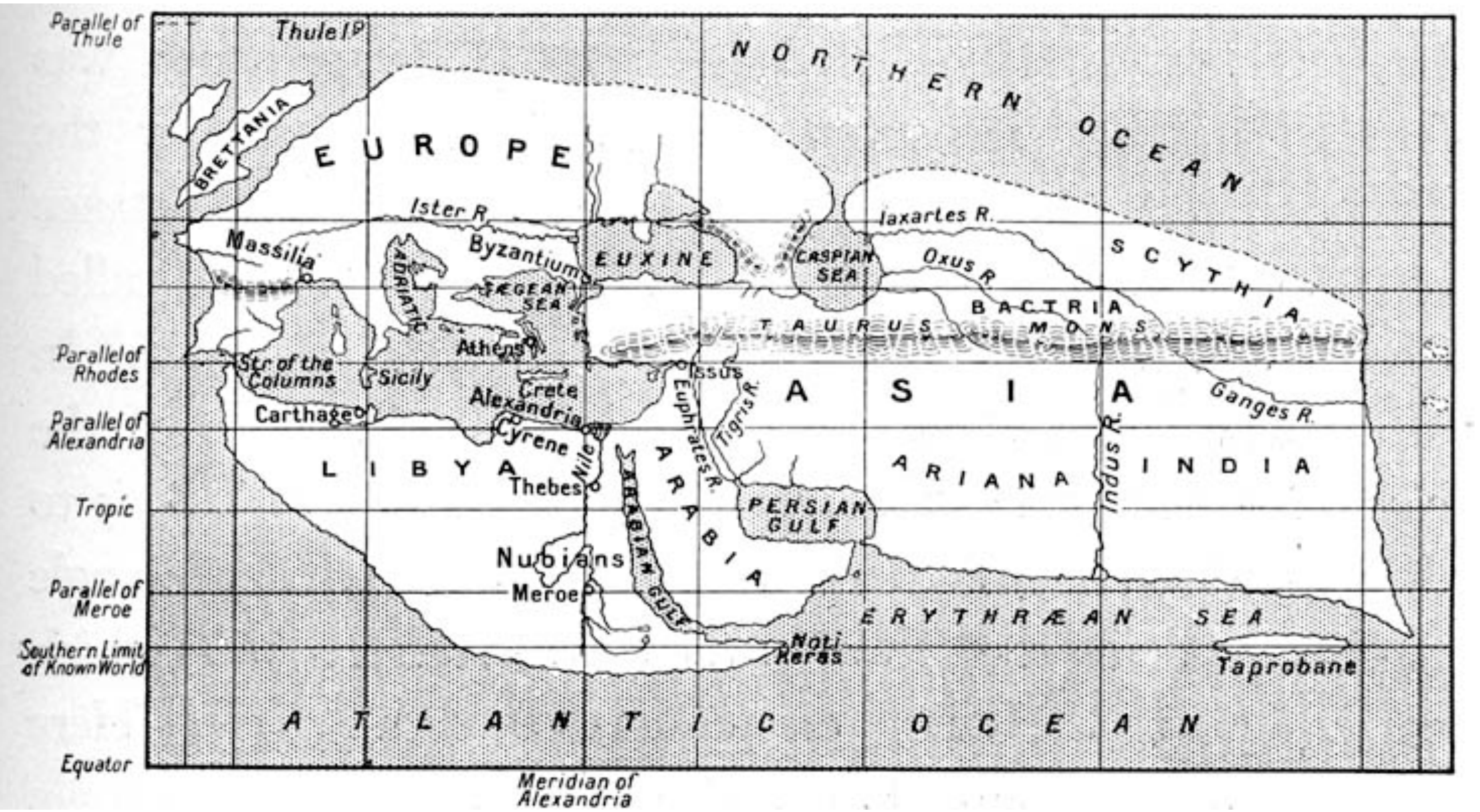
Alexander the Great 356-323 BC – enriched the Hellenistic sciences by science from Egypt, Antioch (todays Turkey) Persia and India. His predecessor in Egypt **Ptolemy I Soter 283–323 BC** and **Ptolemy II Philadelphus 281 – 246 BC** established Alexandria as a knowledge and research world center with a huge library project that not only collected and edited science knowledge and literature, but became home (university-like) for mathematicians, astronomers and other scientists, who were supported by the emperors.

To name some: Euclid, Archimedes, Eratosthenes, Hypatia.

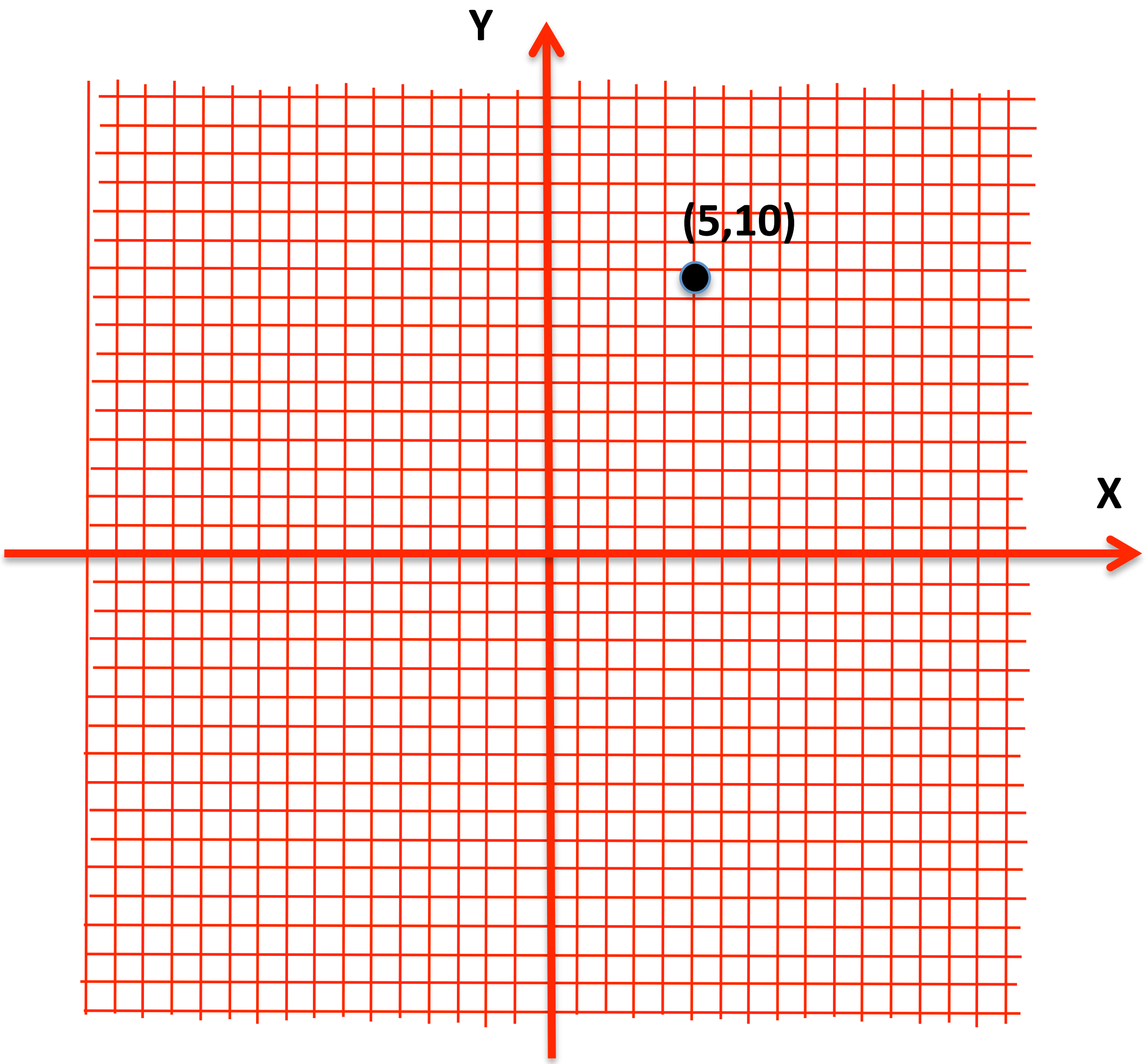
The library in Alexandria caught fire at 47 BC by torches thrown from Julius Caesar's ships in their attack, but was partially recovered.
It was totally burnt down at 640 AC, probably encouraged by Christians and Muslims in the city, after the Arab Caliphate conquered Egypt and Syria from the Byzantine empire. Science was considered a forbidden occupation by religious extremes.

World map by
Fra Mauro, Venice 1460
Ordered by the king of Portugal.
Europe, Asia and Africa are depicted, a coordinate net is drawn.
Cost lines are detailed, but with large scale distortions.
The inside of Asia is not explored and left blank.

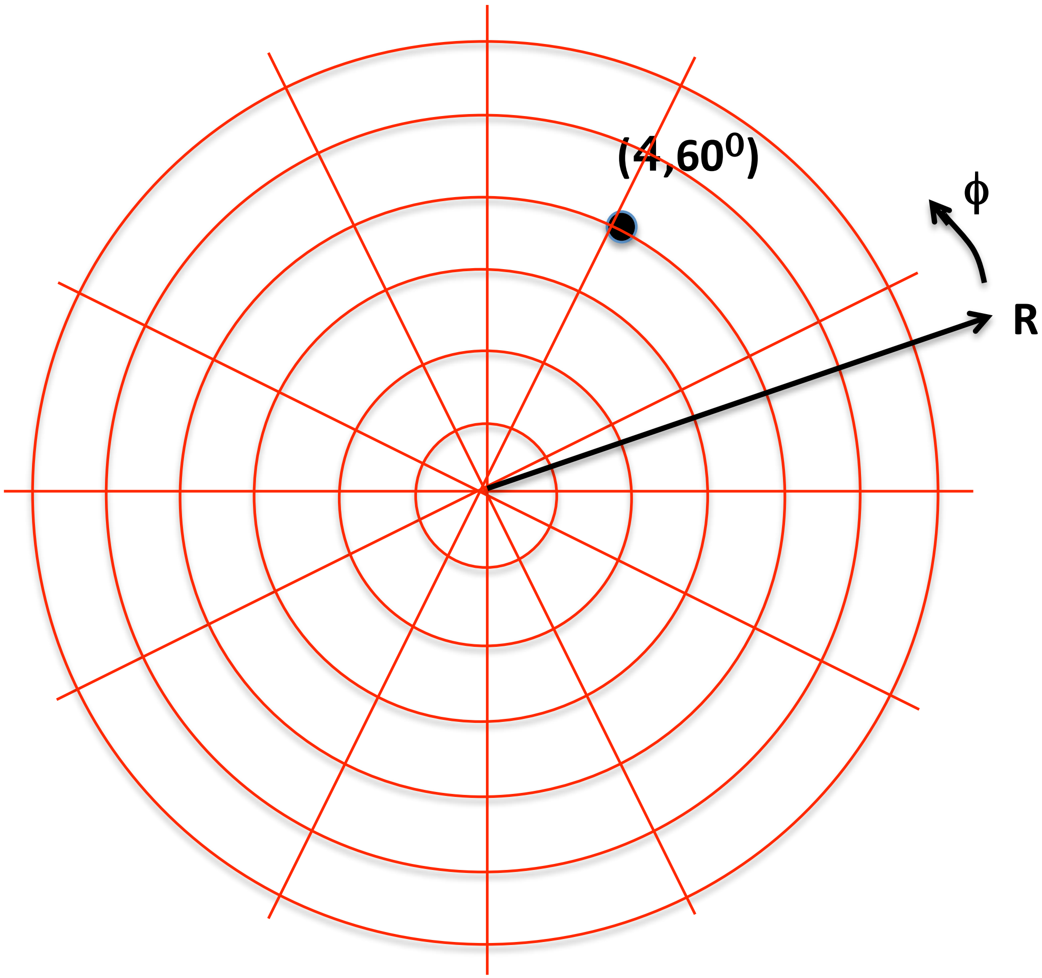
Maps become more detailed with time, and less distorted due to latitude and longitude positioning of sites on the globe.



Aristarchus defined position by horizontal and vertical (Cartesian) coordinates.
The origin X_o, Y_o is arbitrary. Transformation to new origin: $X' = X - X_0$ $Y' = Y - Y_0$



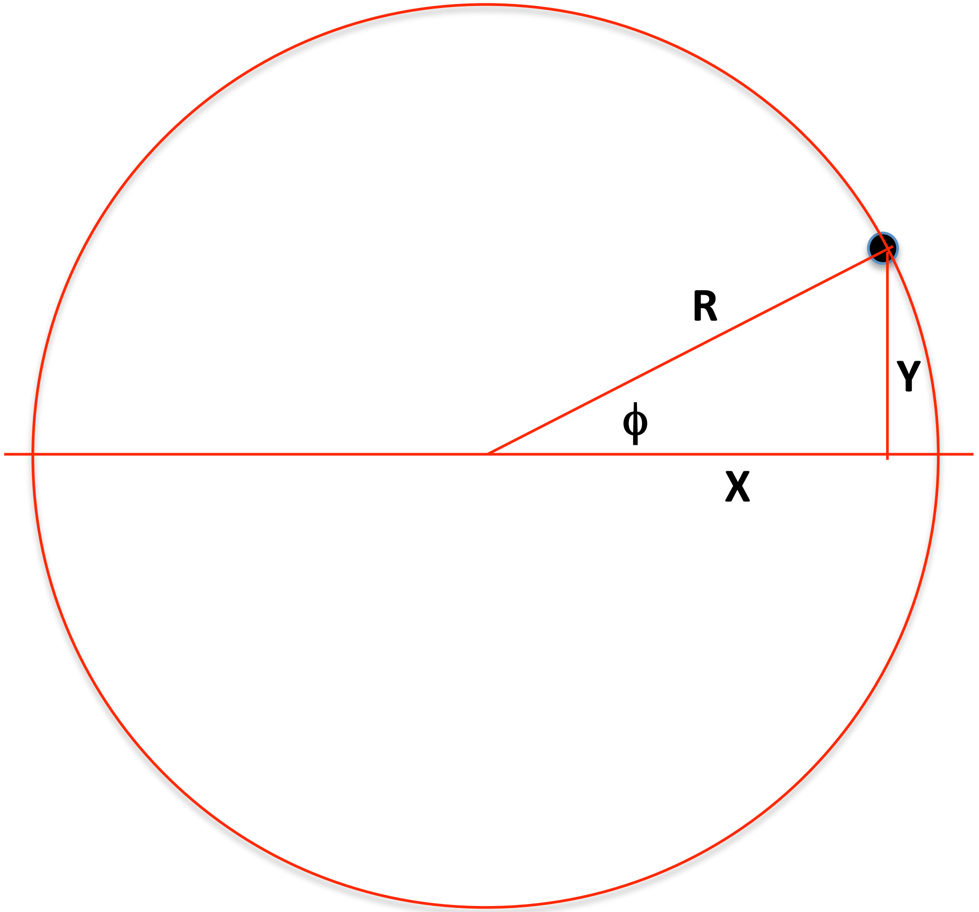
Another set of coordinates: **POLAR**
Each point is presented by angle and radius.



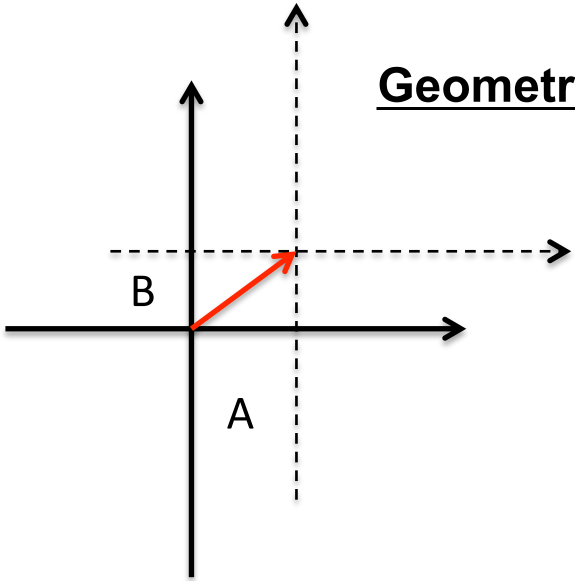
Both systems (Cartesian & Polar) define two independent coordinate in the plane
The transformation between the system (with common origin):

$X = R \cos \phi$ $\quad\quad$ $R = \mathrm{sqrt}(X^2 + Y^2)$

$Y = R \sin \phi$ $\quad\quad$ $\phi = \arctan(Y / X)$



Geometrical transformations:



Shift transformation

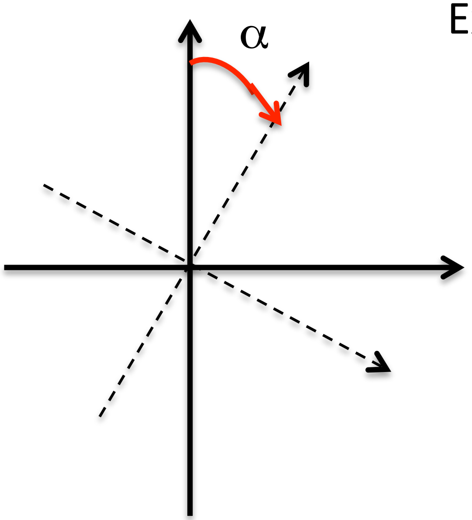
$$X=x+A$$
$$Y=y+B$$

Linear distortion

$$X=ax+by$$
$$Y=cx+dy$$

If $ad-bc = 1 = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

The transformation is a rotation ("orthogonal").
Explicitly, rotation in angle α



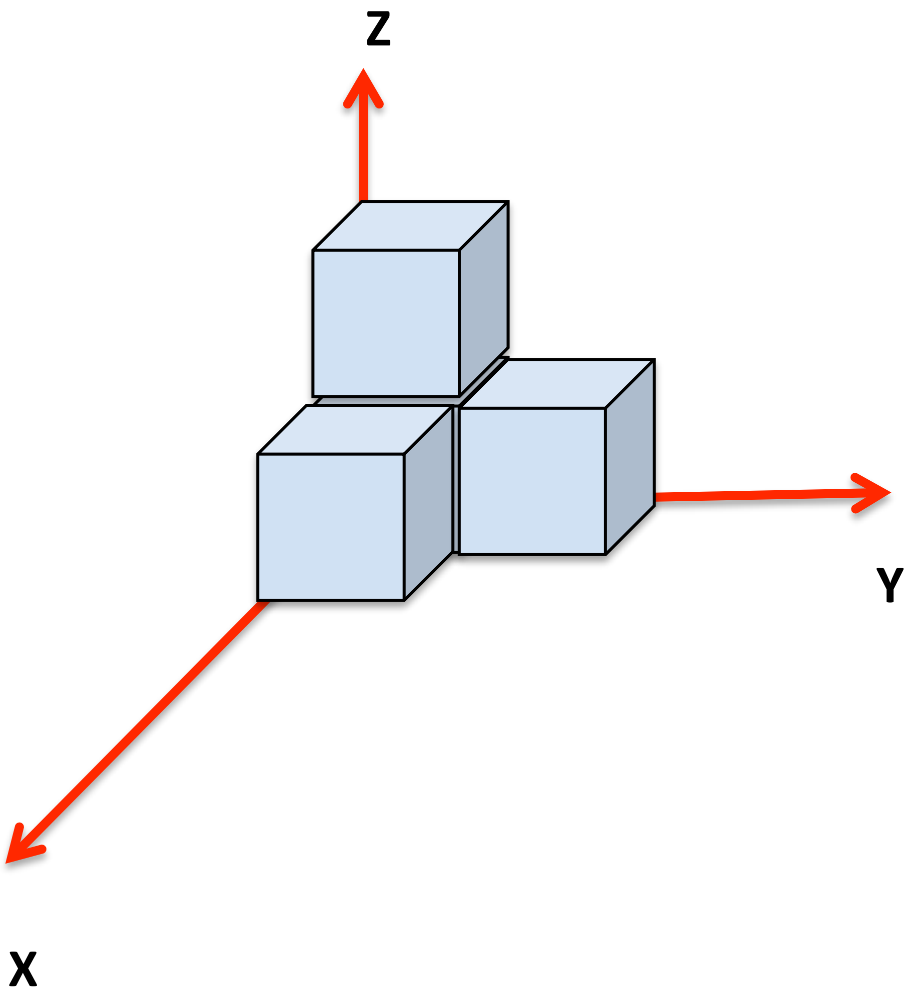
$$X = \cos(\alpha)x + \sin(\alpha)y$$
$$Y = -\sin(\alpha)x + \cos(\alpha)y$$

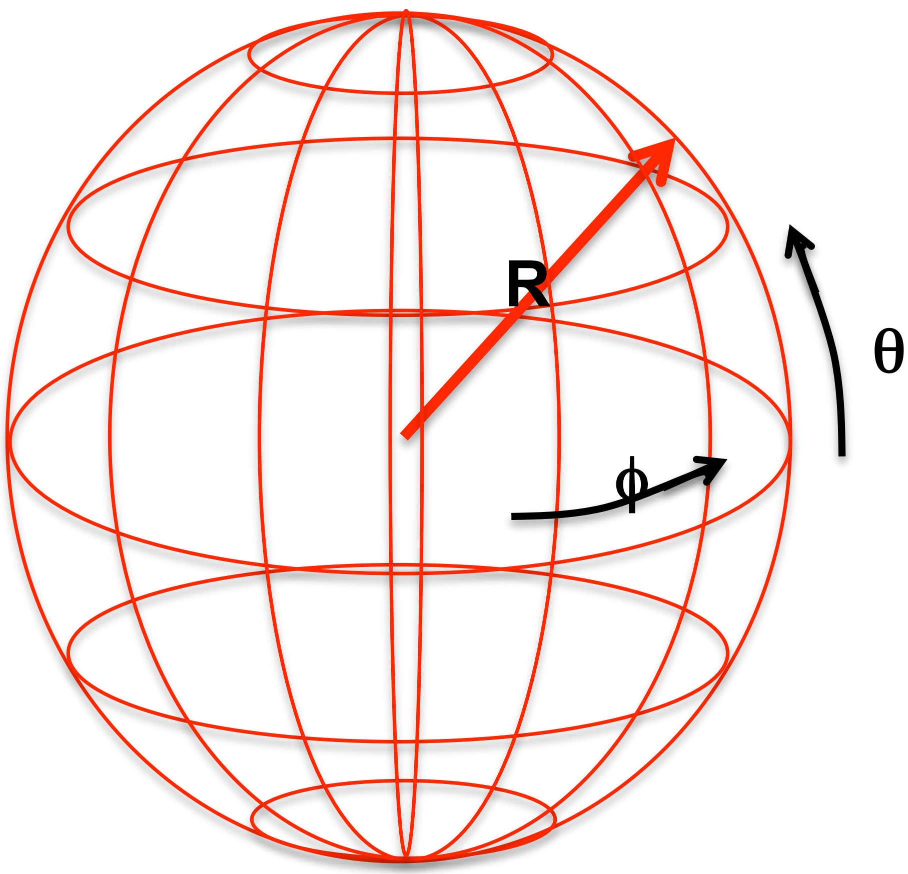
Extension to three-dimensions:

Three Cartesian coordinates: X,Y,Z

Three polar coordinates: R θ φ

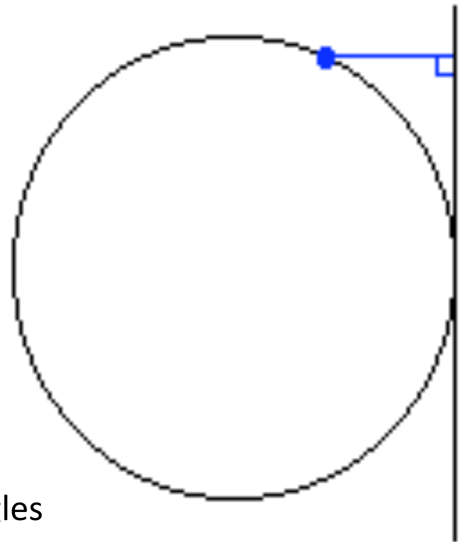
Position of stars defined by θ φ. Radius is not needed (~infinity)





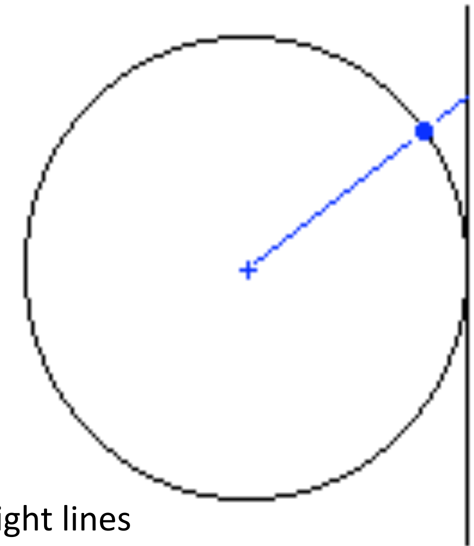
World map – projects a sphere to a plan

Orthographic



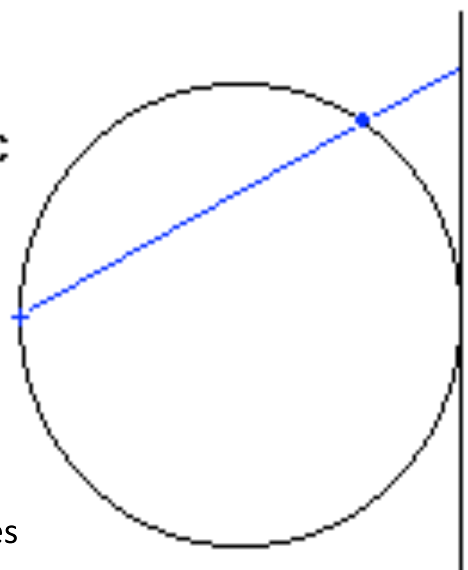
Preserves local angles

Gnomonic



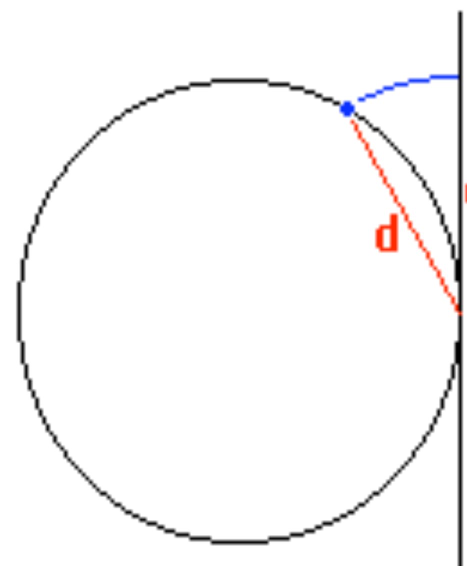
Large circles -> straight lines

Stereographic



Circles -> circles lines->lines

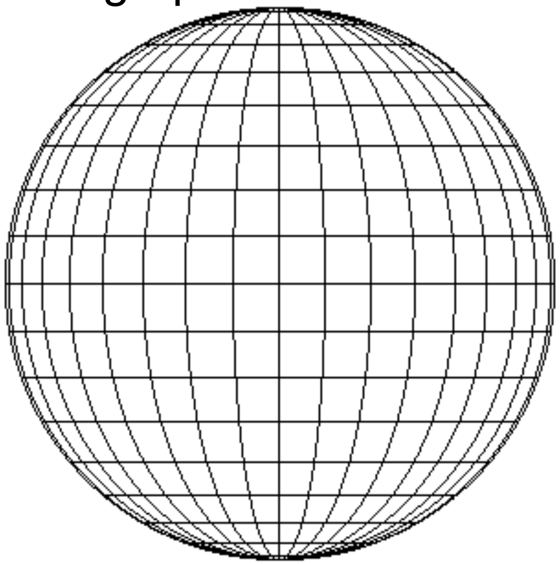
Equal-Area



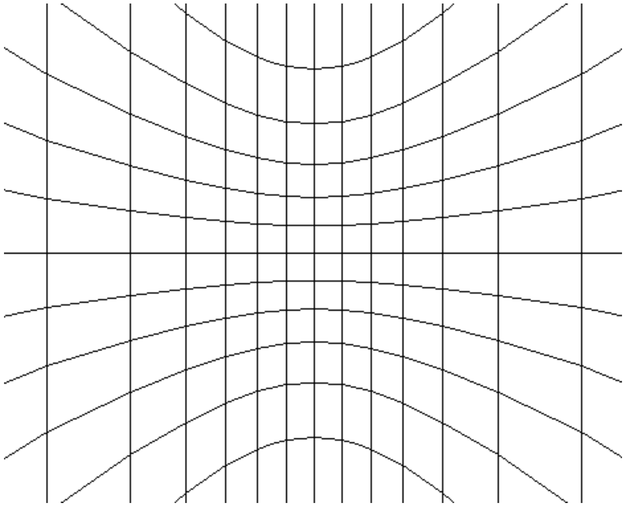
Area preserved

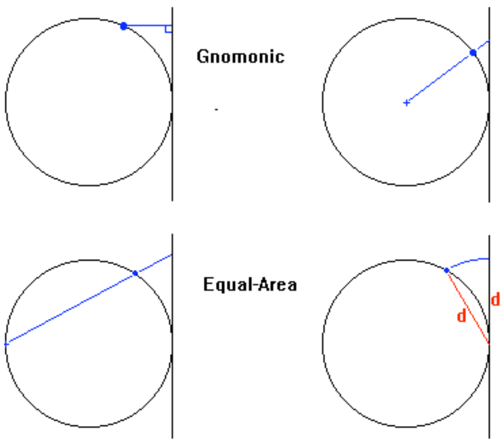
Here is how latitude and longitude lines look for the various projections:

Orthographic

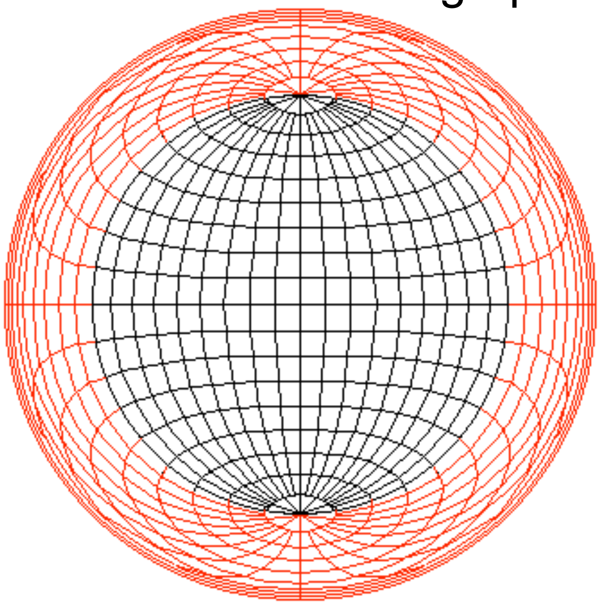


Gnomonic

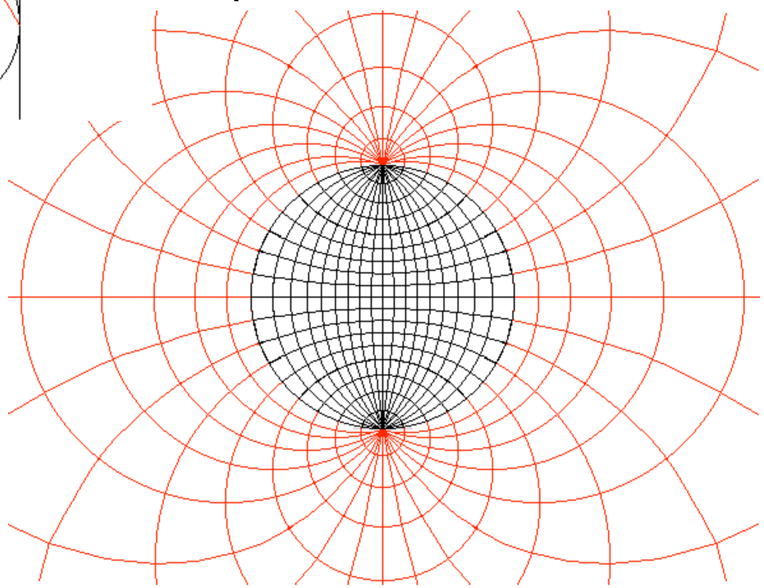




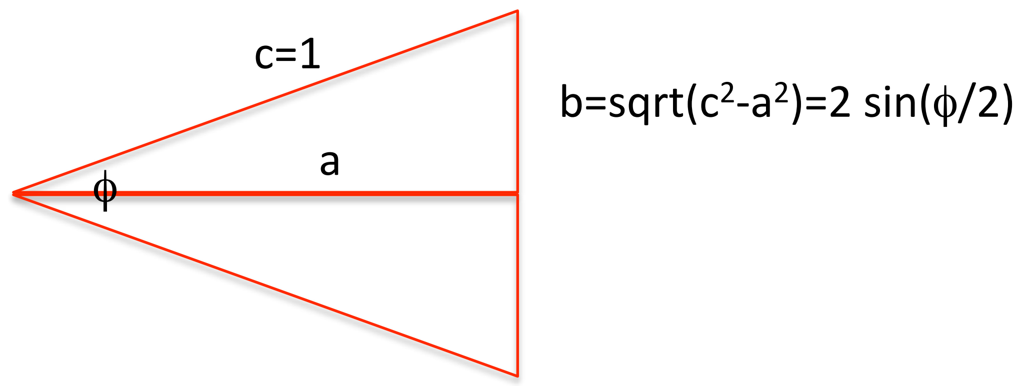
Stereographic



Equal-area

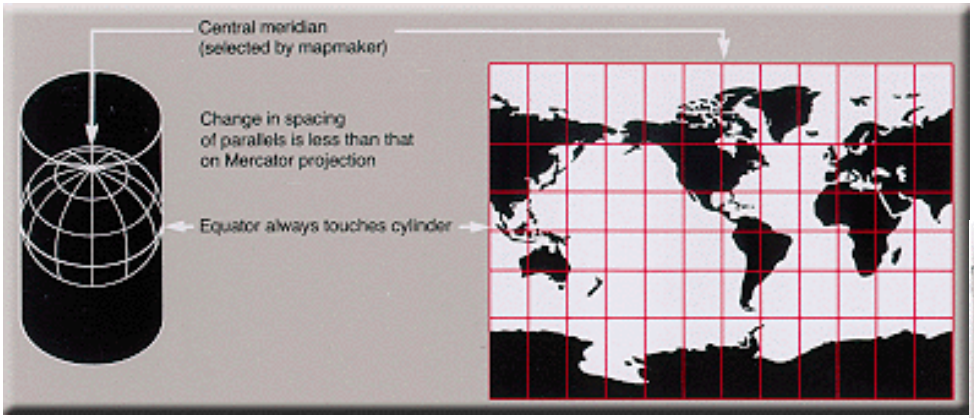


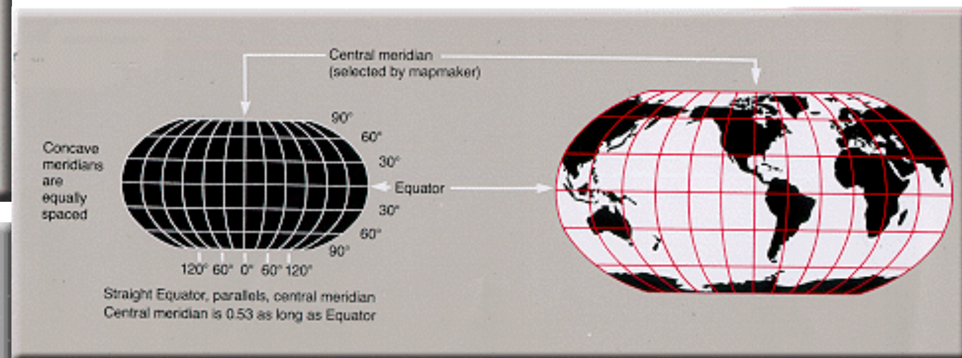
Hipparchus – trigonometric tables (“chord tables”), from calculations of the cords given the orthogonal two edges of right-angle triangles using Pythagoras equation.

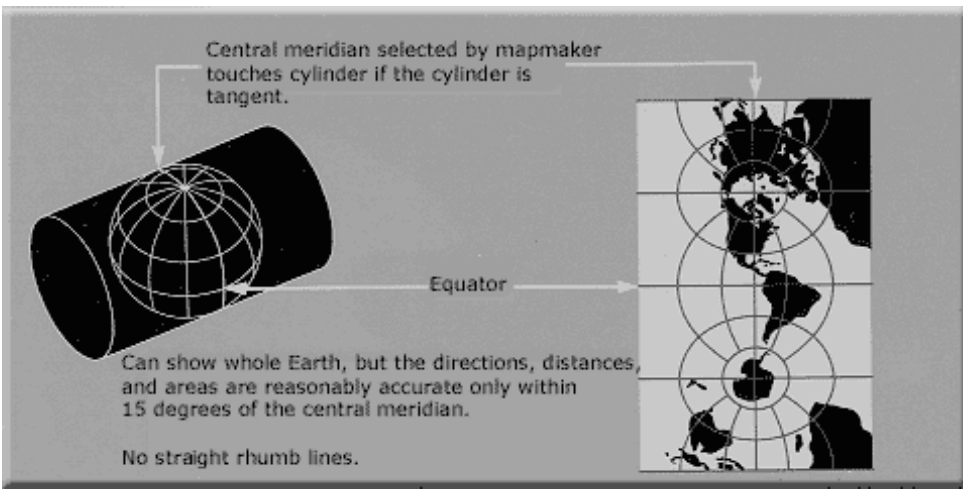


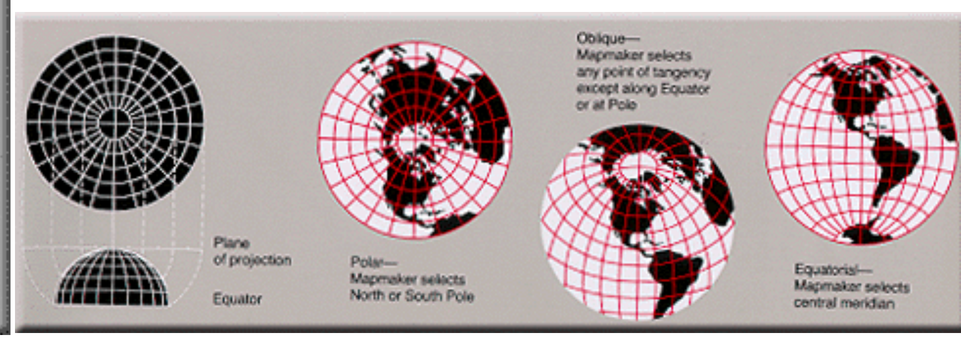
He also show that the projection of a circle the does not cross the center of projection is a circle. This is the basis of Astrolabes, and was used for world geographical mapping. Astrolabes uses in navigation spread out in Arabic ships.
Astrolabes were described by **Al-Biruni.**

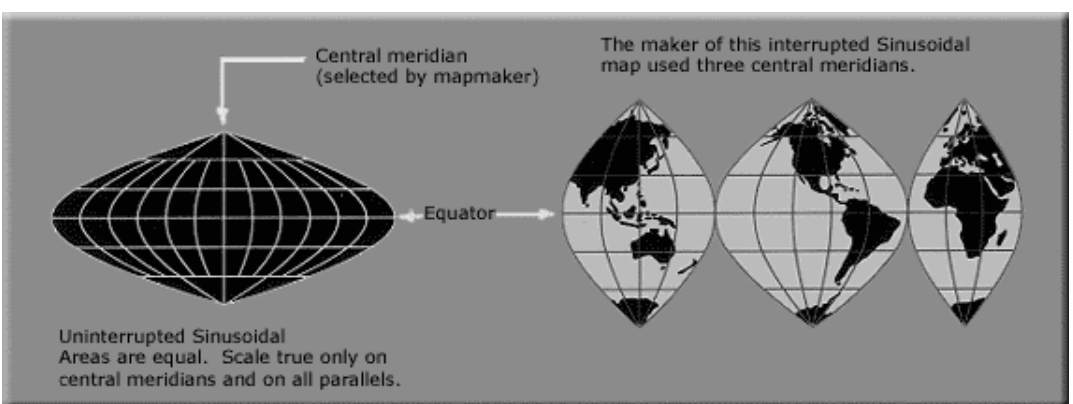
Methods of world projecting maps





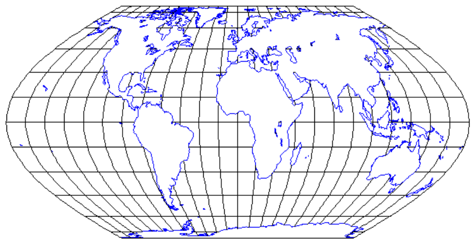






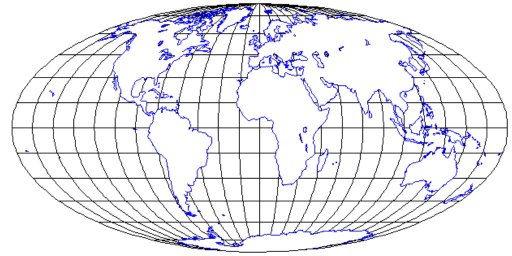
Note the sites of largest distortions

Umbezifferter SANSON-Entwurf



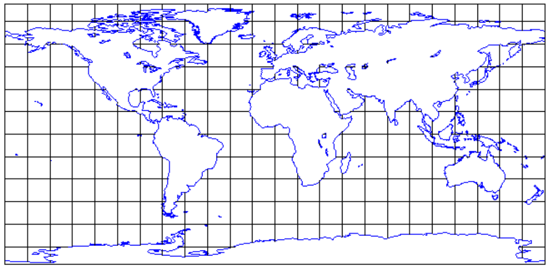
Normale Entwurfsachse (0, 90, 0) Karto 4.5

Entwurf von MOLLWEIDE



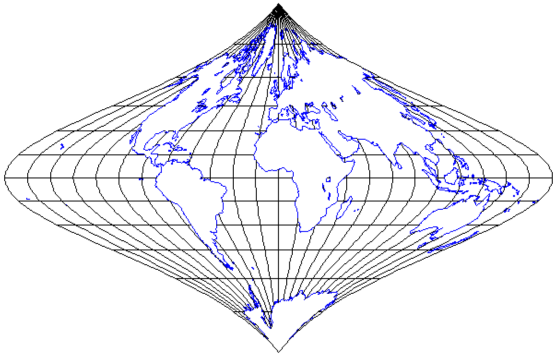
Normale Entwurfsachse (0, 90, 0) Karto 4.5

Abstandstreuer Zylinderentwurf



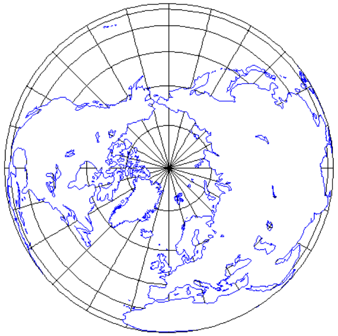
Normale Entwurfsachse (0, 90, 0) Karto 4.5

Entwurf von PREPETIT-FOUCAULT



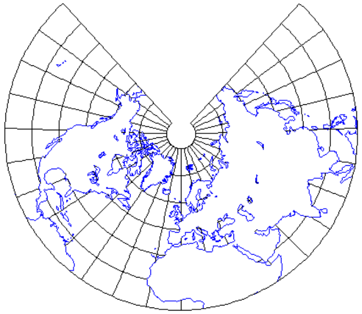
Normale Entwurfsachse (0, 90, 0) Karto 4.5

Orthographische Projektion



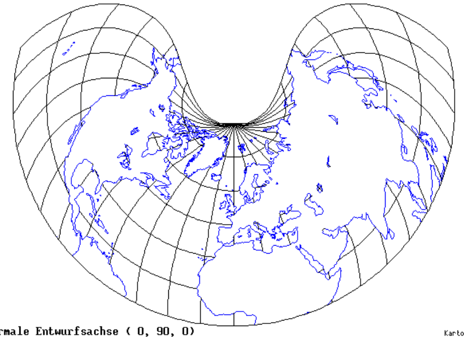
Normale Entwurfsachse (0, 90, 0) Karto 4.5

Abstandstreuer Kegelentwurf



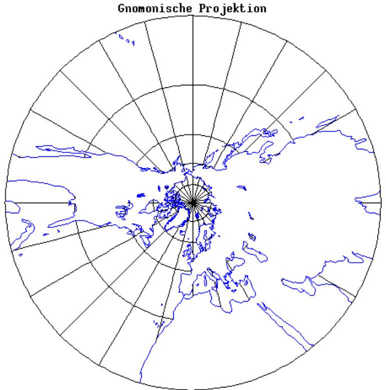
Normale Entwurfsachse (0, 90, 0) Karto 4.5

Entwurf von BONNE



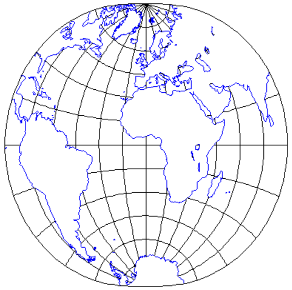
Normale Entwurfsachse (0, 90, 0) Karto 4.5

Gnomonische Projektion



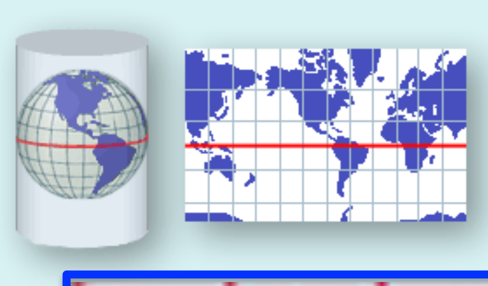
Normale Entwurfsachse (0, 90, 0) Karto 4.5

Polykonischer Entwurf



Normale Entwurfsachse (0, 90, 0) Karto 4.5

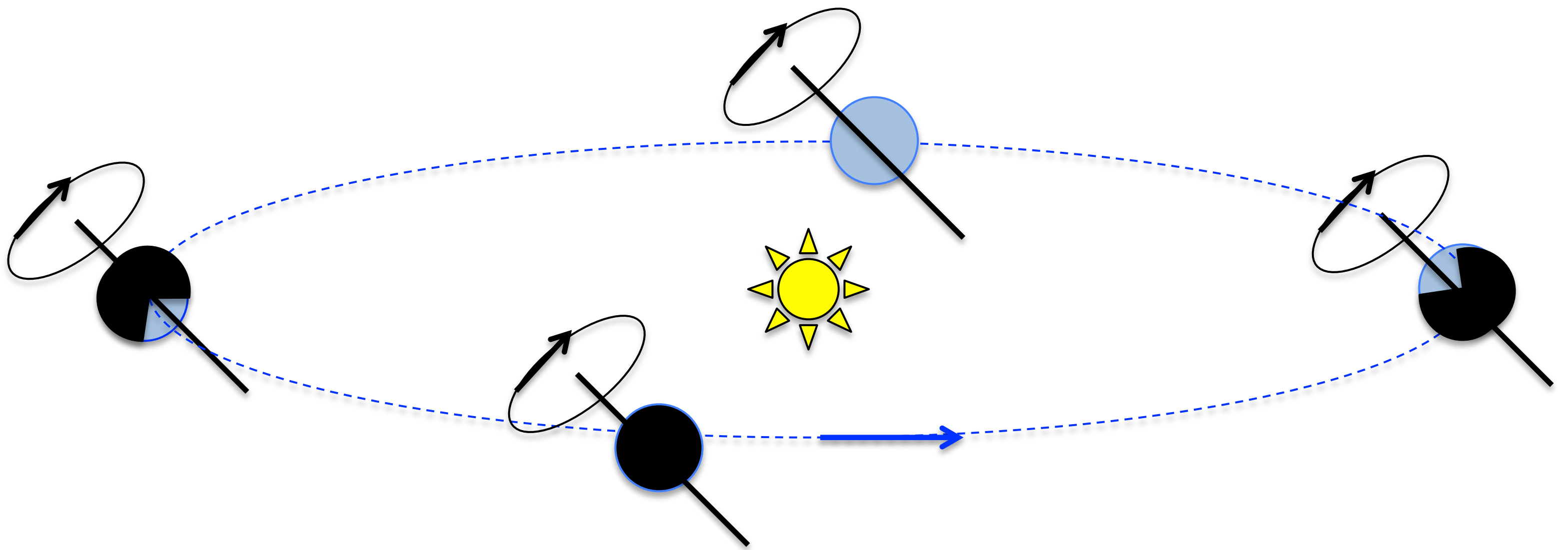
http://www.geometrie.tuwien.ac.at/karto/



The shape of the border between day and night (half sphere)

Mercator projection map

Polar circles – latitude beyond which half year day, half year night



Appendix: The solar system

Planet	Distance From Sun (km)	No of Years of Travel from the Sun (yrs)	Distance From Earth (km)	No of Years of Travel From Earth(yrs)
Mercury	58,000,000	13	92,000,000	21
Venus	108,000,000	25	42,000,000	10
Earth	150,000,000	34	-	-
Mars	228,000,000	52	78,000,000	18
Jupiter	778,000,000	178	628,000,000	143
Saturn	1,427,000,000	326	1,277,000,000	292
Uranus	2,870,000,000	655	2,720,000,000	621
Neptune	4,497,000,000	1,027	4,347,000,000	992
Pluto*	5,870,000,000	1,340	5,720,000,000	1,306

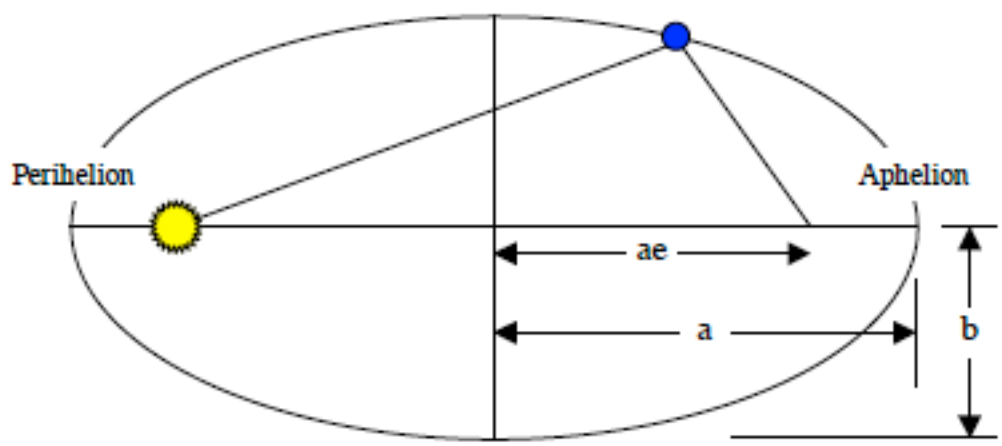
Eccentricity of planet orbits

$e = \sqrt{1 - b^2/a^2}$

Planet	Semi-major axis(AU)	Sidereal period of revolution (d,y)	Period of rotation (d,h)	Eccentricity	Inclination of orbit to the ecliptic (°)	Average orbital speed (km/s)
Mercury	0.3871	87.969 *d*	58.646 d	0.206	7.00	47.9
Venus	0.7233	224.70 *d*	243.01[R] d	0.0068	3.39	35.0
Earth	1.0000	365.256 *d*	1.000 d	0.017	0.00	29.77
Mars	1.5236	686.98 *d*	1.026 d	0.093	1.85	24.1
Jupiter	5.2026	11.856 *y*	9.936 h	0.048	1.30	13.1
Saturn	9.5719	29.369 *y*	10.656 h	0.053	2.48	9.64
Uranus	19.194	84.099 *y*	17.232[R] h	0.043	0.77	6.83
Neptune	30.066	164.86 *y*	16.104 h	0.010	1.77	5.5
Pluto*	39.537	248.60 *y*	6.387[R] d	0.250	17.12	4.7

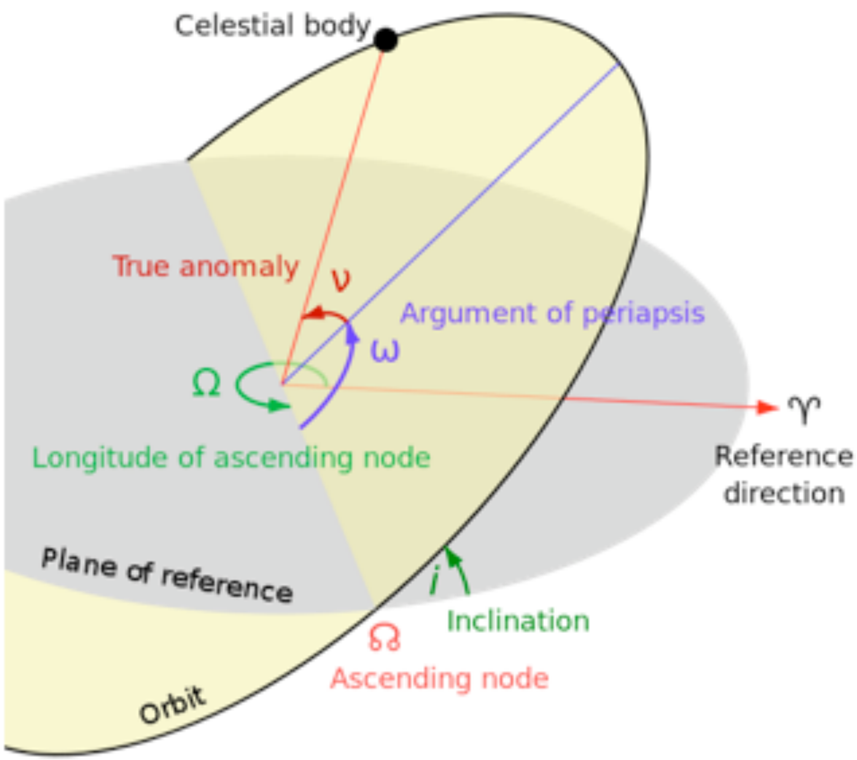
R = retrograde

* Pluto has been reclassified as a dwarf planet (Section 3.1).

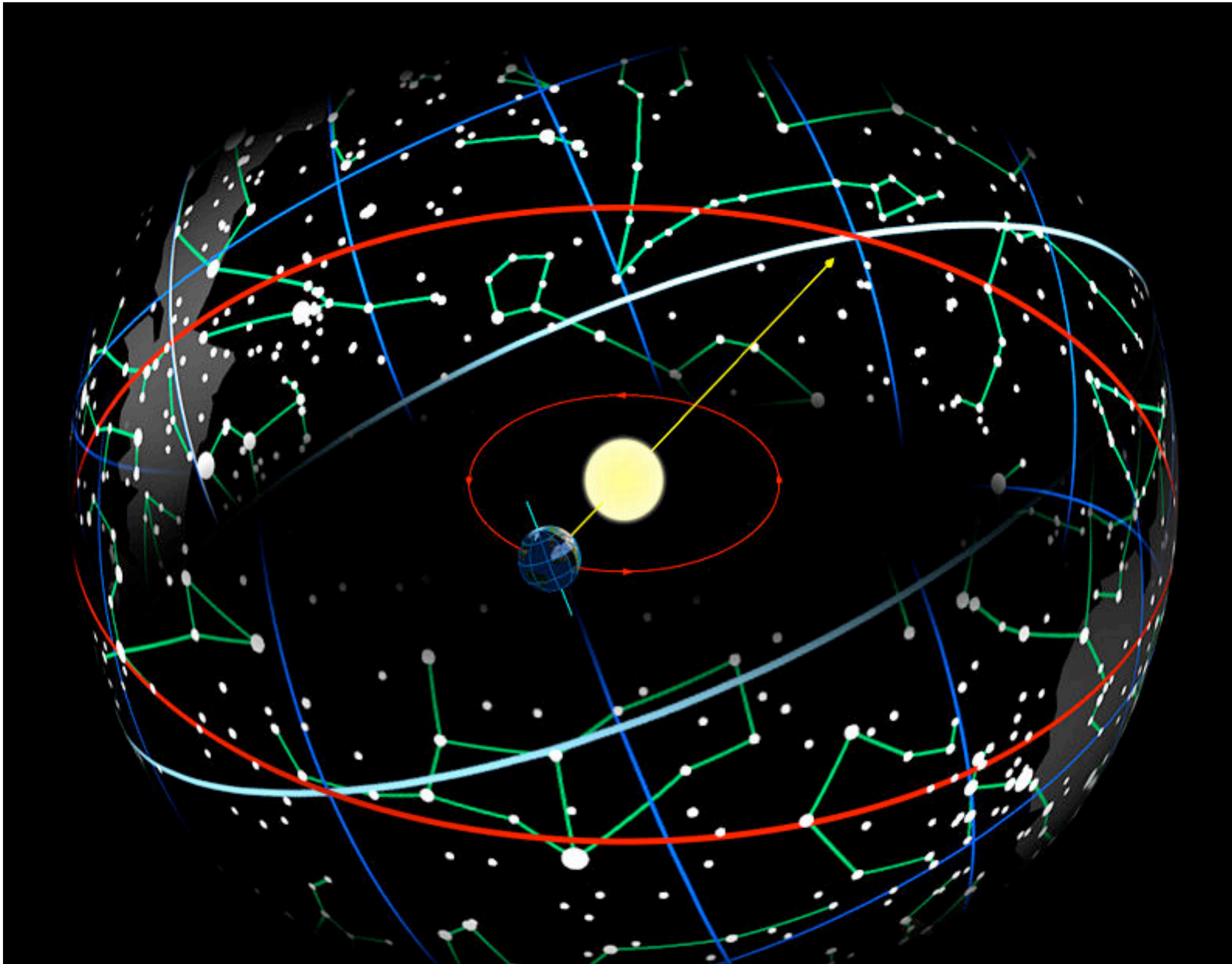


Inclination of planet orbits

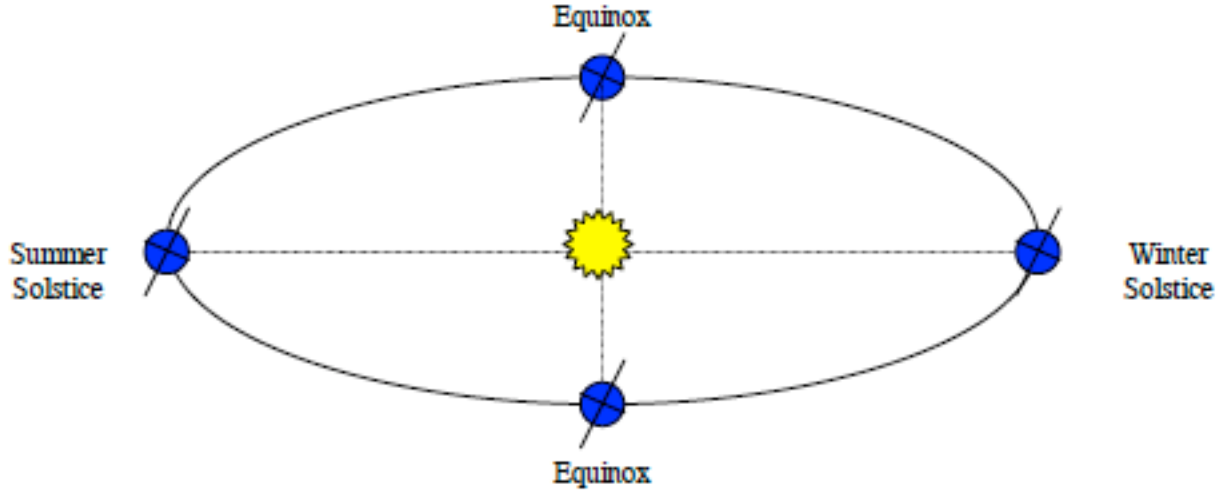
	Mercury	Venus	Earth	Mars	Jupiter	Saturn	Uranus	Neptune	Pluto
Mass (kg)	3.3×10^{23}	4.9×10^{24}	6.0×10^{24}	6.4×10^{23}	1.9×10^{27}	5.7×10^{26}	8.7×10^{25}	1.0×10^{26}	1.3×10^{22}
Radius (km)	2,439	6,051	6,378	3,393	71,492	60,268	25,559	24,764	1,150
Inclination	2^{o}	$177^{o}.30$	$23^{o}.45$	$25^{o}.45$	$3^{o}.12$	$26^{o}.73$	$97^{o}.86$	$29^{o}.60$	$122^{o}.46$
Period (yr)	0.24	0.62	1.00	1.88	11.86	29.46	84.01	164.97	248.40
Radius (AU)	0.39	0.72	1.00	1.52	5.20	9.54	19.19	30.06	39.53
Eccentricity	0.206	0.007	0.017	0.093	0.048	0.055	0.047	0.009	0.249
Sun/planet mass	6023600	408523.5	332946.038	3098710	1047.350	3498.0	22960	19314	130000000

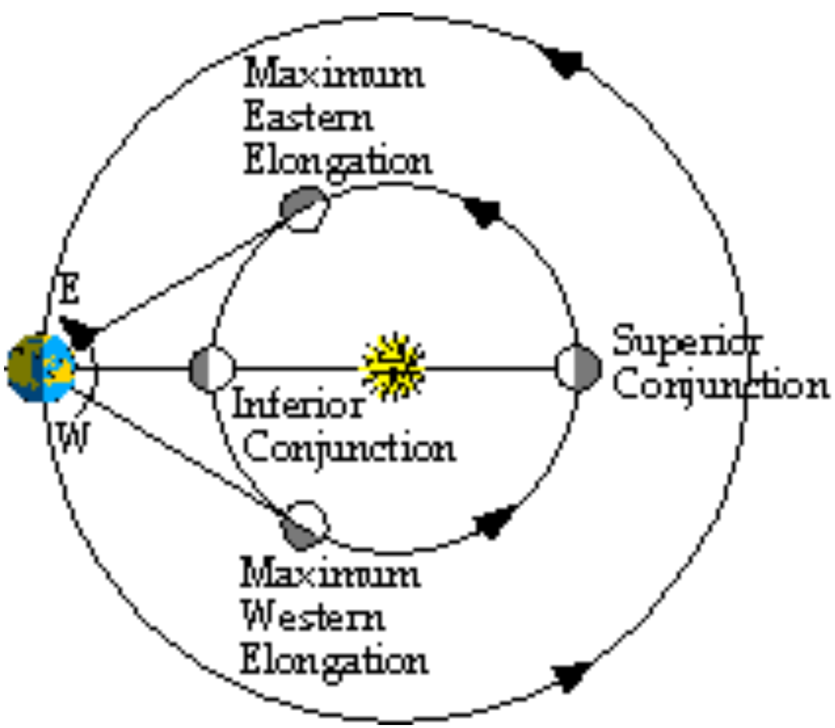


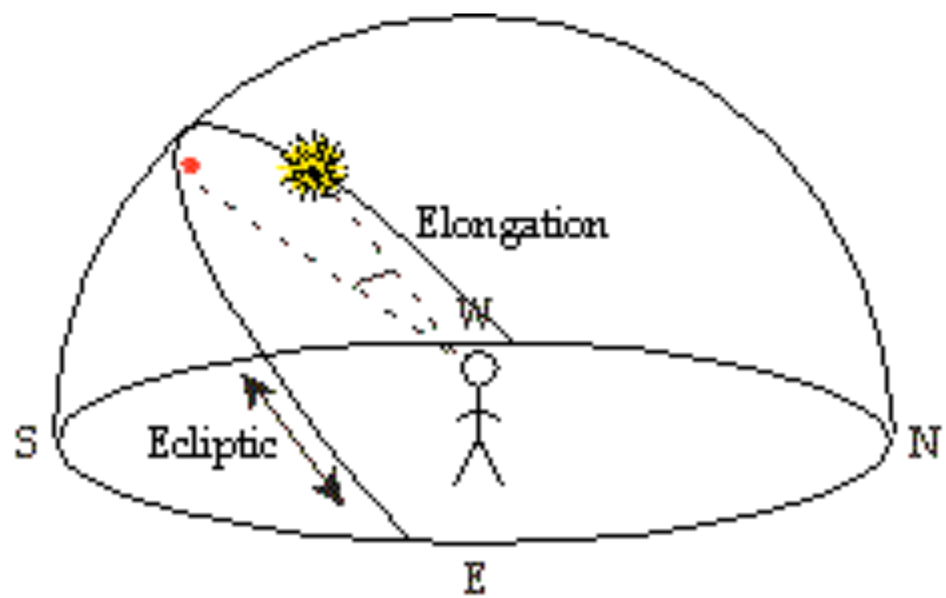
The sun seems rotating in the eclipse plane (red) that is tilted vs. earth rotation (blue)



NOTE: 1 AU = 14.96×10^7 km







SUMMARY:

Astronomy in the ancient and classical era was a field of intense interest to scientists. The applications to religion, agriculture and navigation (in space or time) turned astronomy into the force motive for better star measurements and development of advanced mathematical modeling. Development of Geometry and Trigonometry, (including spherical Trigonometry) was tightly linked with improved measurements of star positions.

The apparent circular motion of the sun and moon around earth, and the complex motion of the planets relative to stars, required complex theories that proposed different laws for (circular) star motion than for earthy objects (that move in straight lines). To keep stars up in the sky, crystal spheres were invented. To explain the course of planets, epicycles were described, and all these artificial inventions persisted for thousands of years, due to the strong biased belief in the central status of humans in the universe.